Geometric Modeling

Solid Modeling

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www.povcomp.com/hof/Villarceau_Circles-CSG.html

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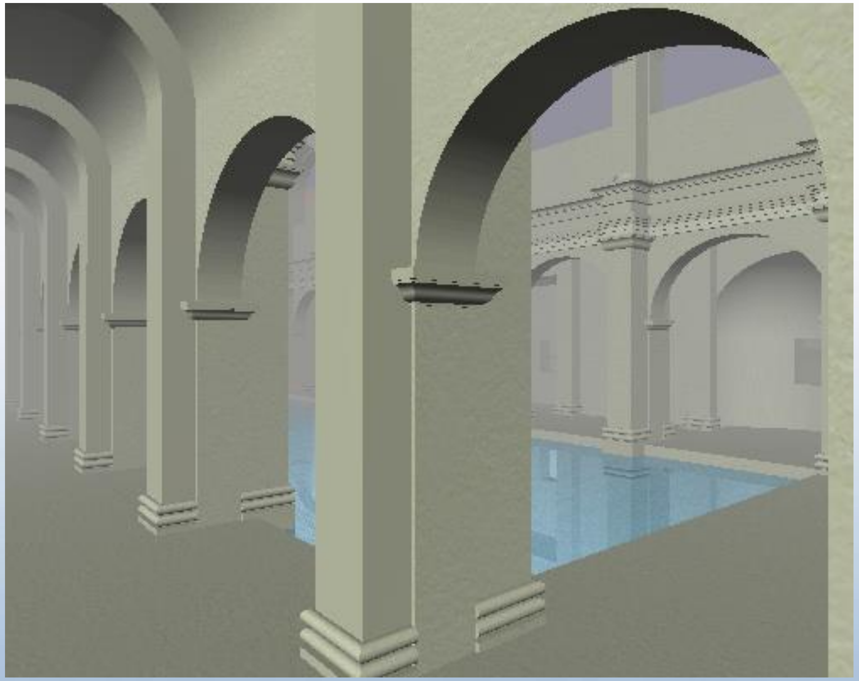

The svLis model of the Great Bath in in Aquae Sulis as it was in 200 AD

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"Still with Bolts" by Jaime Vives Piqueres (2002)

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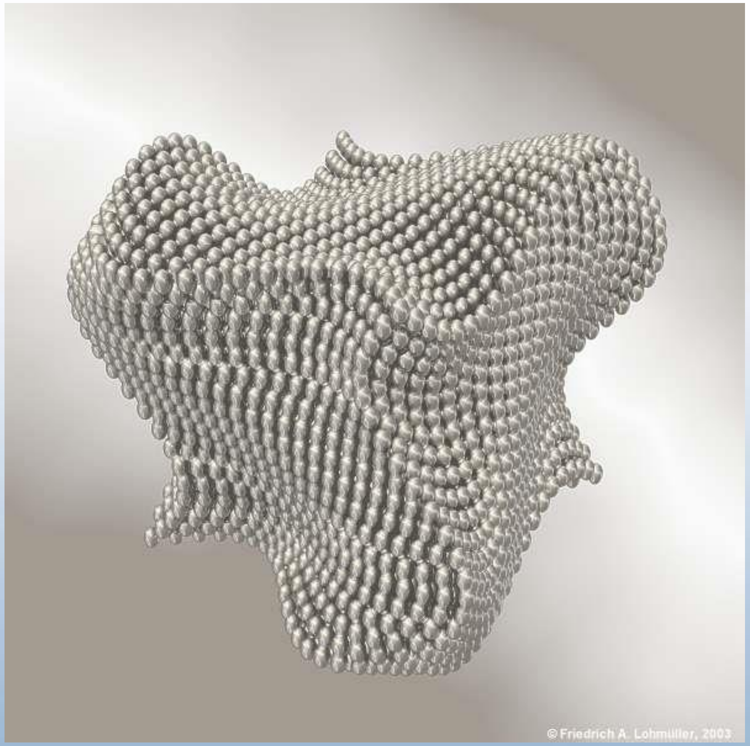
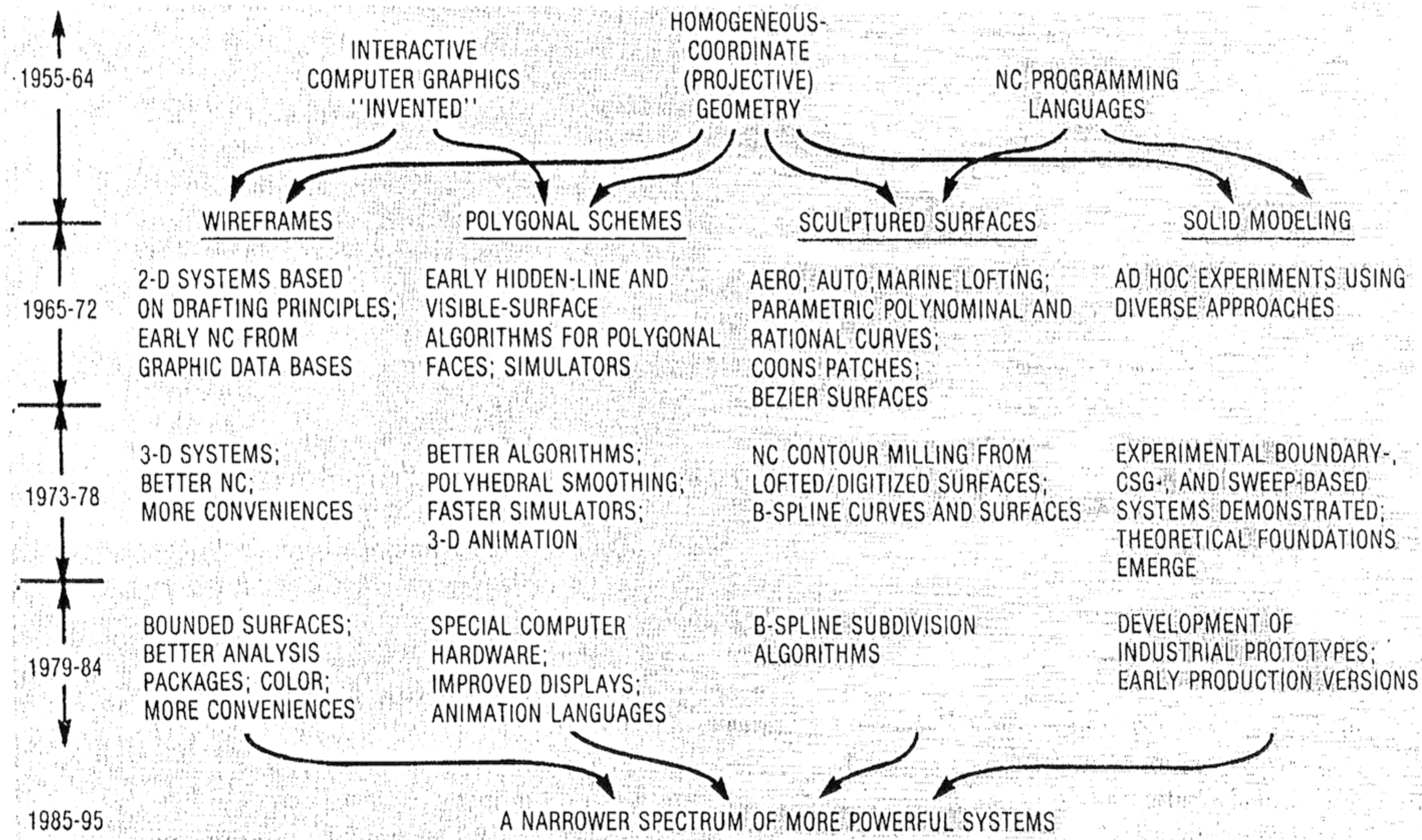


"Dancing Cube" by Friedrich A. Lohmueller (2003)

A. A. G. Requicha, H. B. Voelcker, 1982

A historical summary of approaches to 3-D object representation

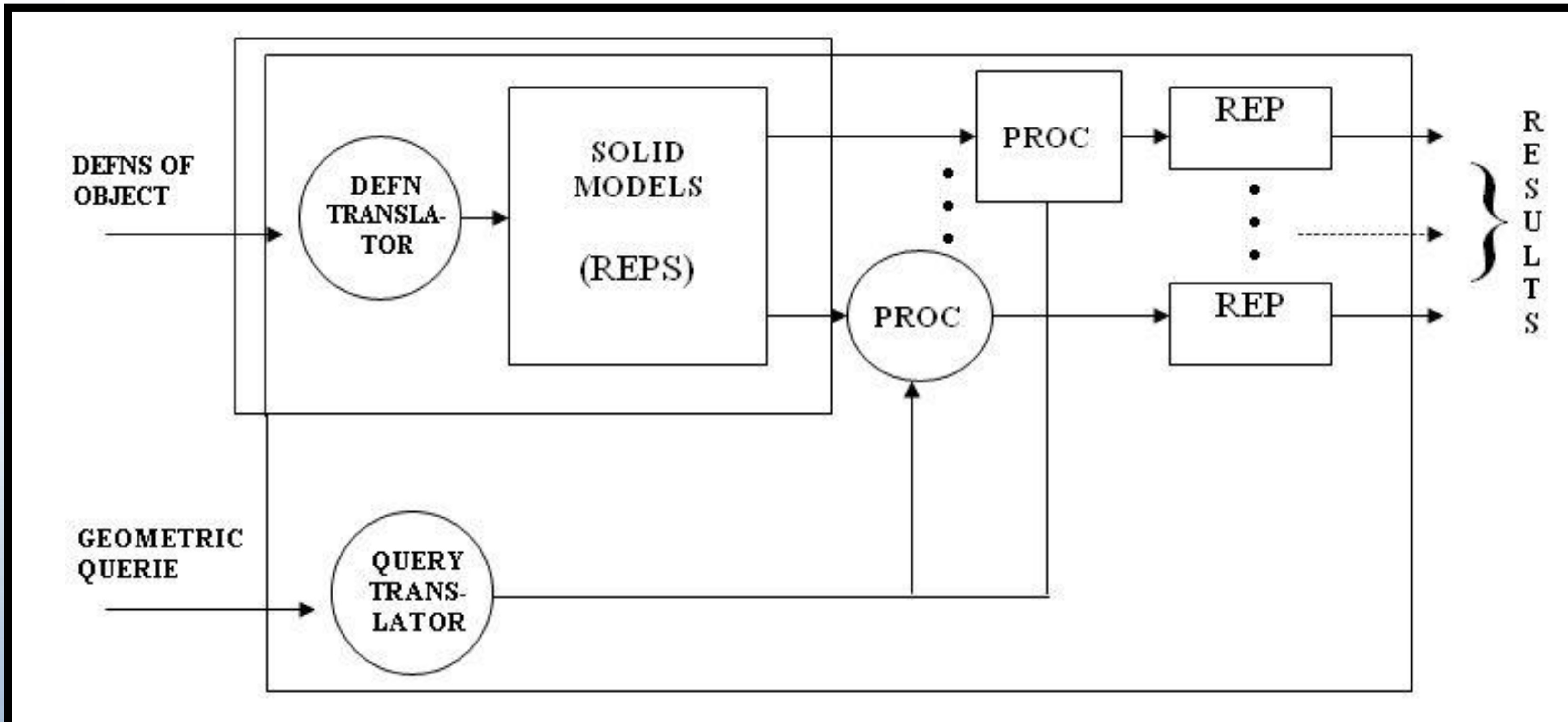


Solid Modelling System

A solid modelling system has four primary components [Requicha, 1980]:

- Symbol structures which represent solid objects;
- Processes which answer geometric questions (such as “What is the volume?”) using the symbol structures;
- Input facilities for creating and editing object representation;
- Output facilities and representations of result.

Solid Modelling System



Requicha, Comp. Surveys, p.438

Solid modelling system is a subsystem which provides entering, storing and modifying object representation.

Abstract solid

Properties that should be captured by the notion of "abstract solid":

- Rigidity: An abstract solid must have an invariant shape which is independent of the solid's location and orientation.
- Homogeneous three dimensionality: A solid must have an interior, and a solid's boundary cannot have isolated or dangling portions.
- Finiteness: A solid must occupy a finite portion of space.

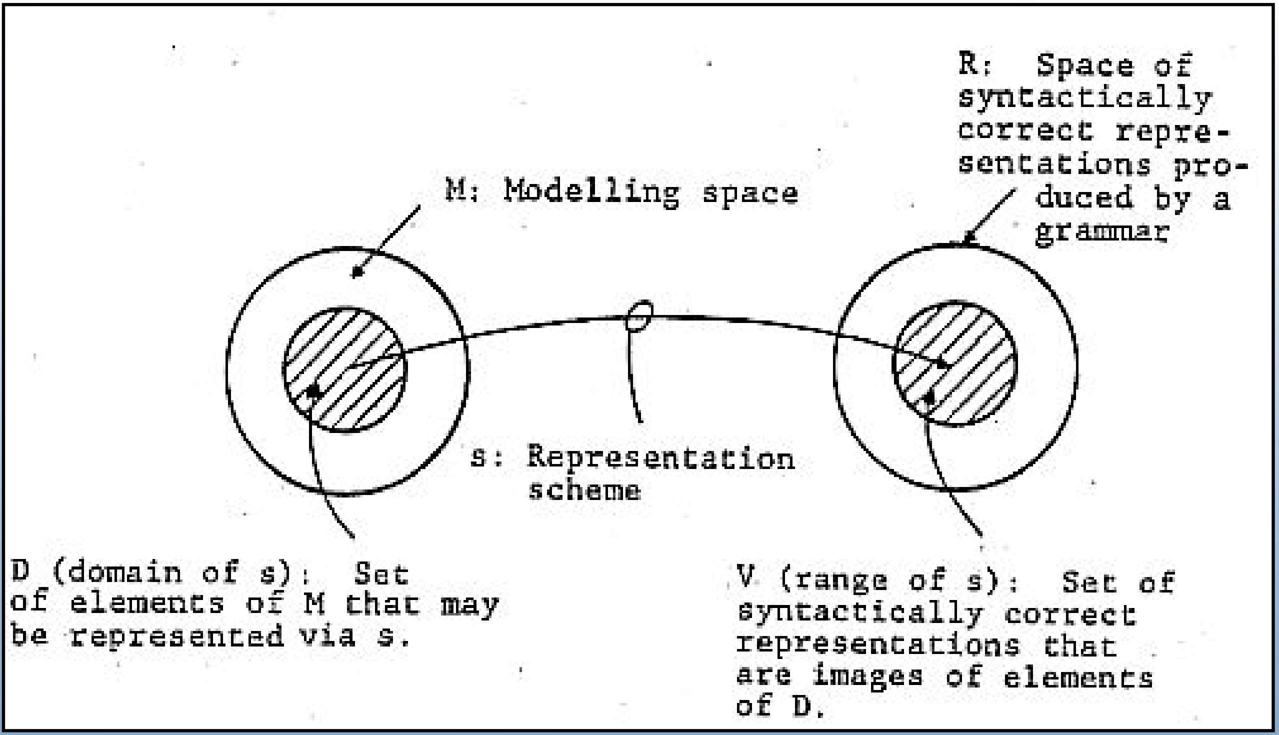
Abstract solid

- Closure: Rigid motions (translations and rotations) or operations that add or remove material (set-theoretic operations) must produce another solid.
- Finite describability: There must be some finite aspect of 3D models of solids (a finite number of “faces”) to make them representable in computers.
- Boundary determinism: The boundary of a solid must determine unambiguously what is “inside” the solid.

A representation scheme establishes a correspondence between *M* and *R*.

It is defined as relation

s: M → R



Requicha, Comp. Surveys, p.441

Abstract solid : r-sets

Suitable models for solids are r-sets that are bounded, closed, regular, and semi analytic subsets of 3D Euclidean space (E^3).

- A bounded set

  occupies a finite portion of space.

- Closed set

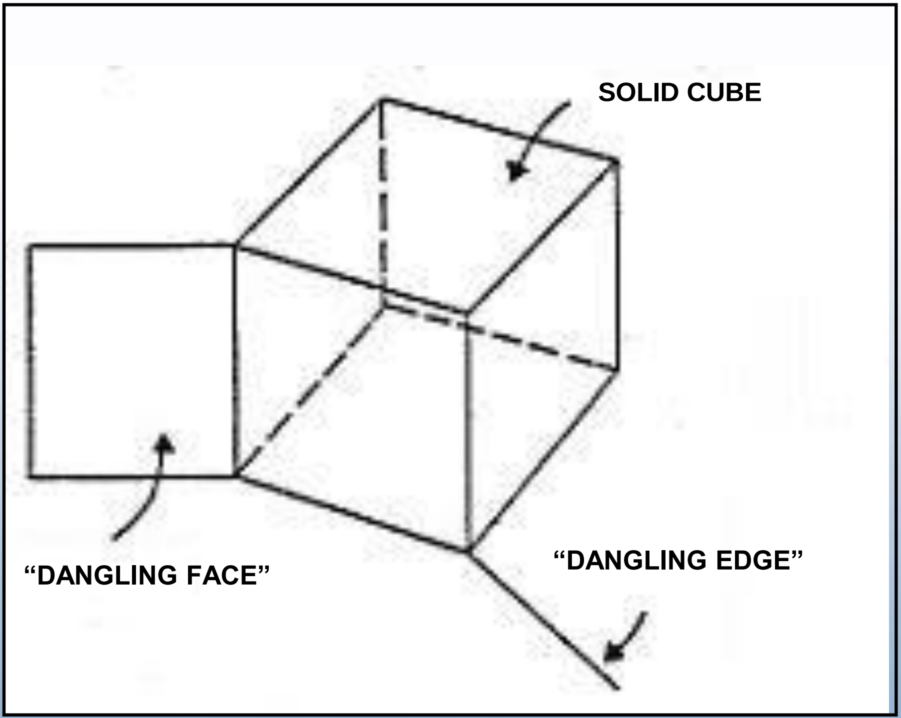
  The set is closed if it contains its boundary.

Abstract solid : r-sets

- Reqular

It is a closed set. It contains its boundary.

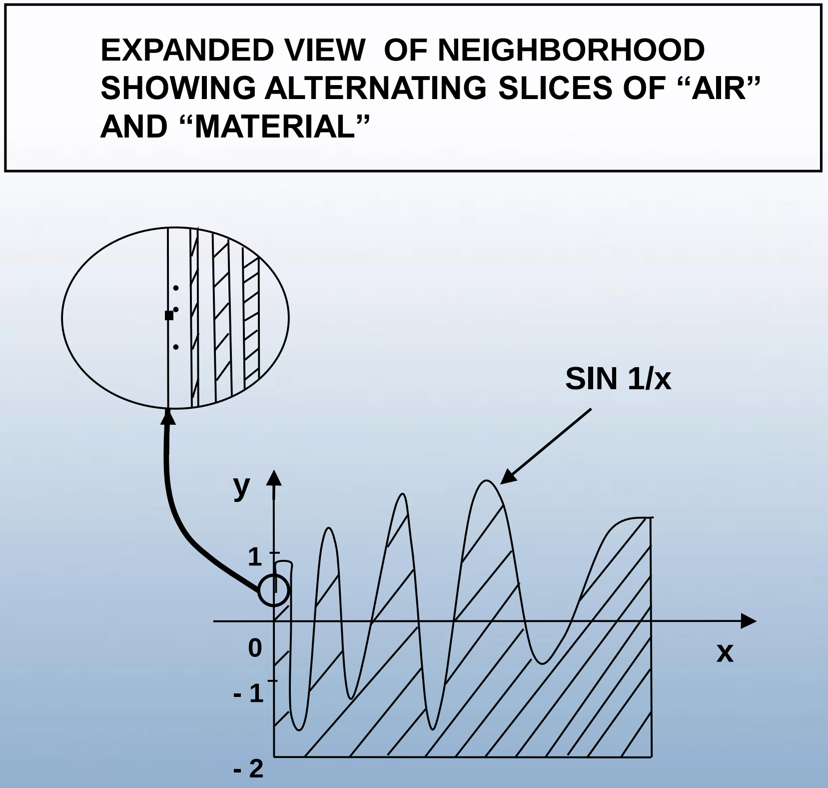
It is not a regular set because its boundary has dangling portions that are not adjacent to the set's interior.



Requicha, Comp. Surveys, p.440

- Semi-analytic set

This set is not semi-analytic because its top face is ill-behaved; it oscillates infinitely as it approaches the left face.



Requicha, Comp. Surveys, p.440

Representation scheme

Syntactically correct representations are finite symbol structures constructed according to syntactical rules.

- The collection of all syntactically correct representations is called a representation space *R*.
- Abstract solids (r-sets) are the elements of a mathematical modelling space M.

Formal properties of representations

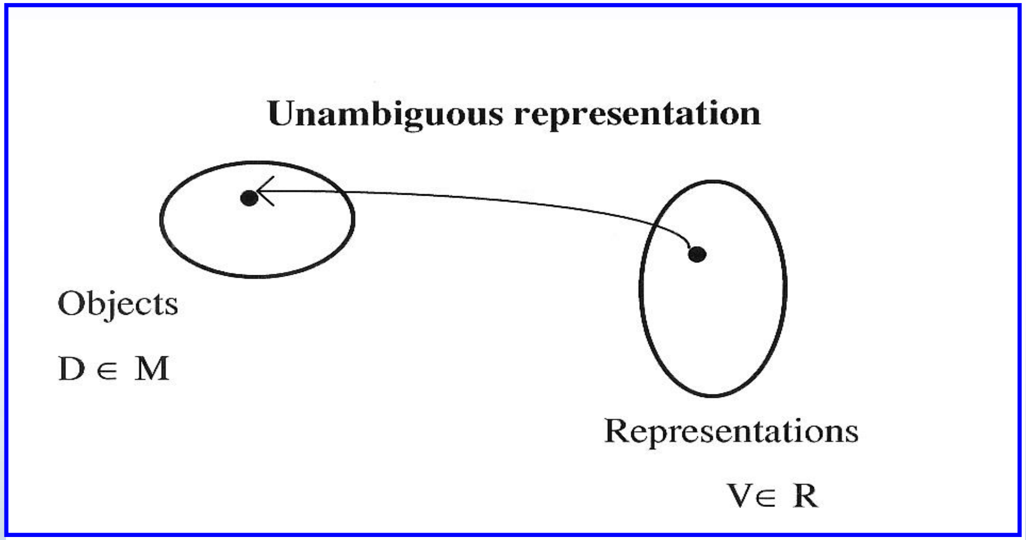
- Domain

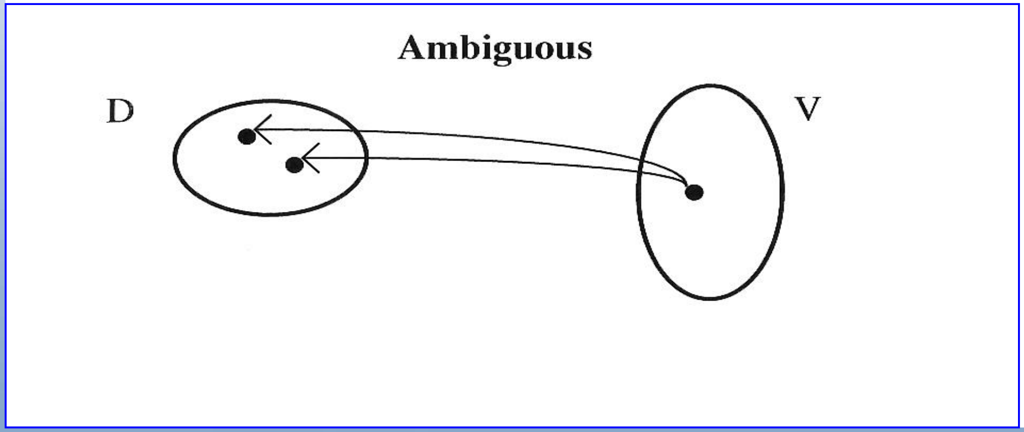
  The domain is the set of solids representable in the representation scheme (its descriptive power).

- Validity

  A symbol structure, which corresponds to a nonsense object, should not exist.

Formal properties of representations





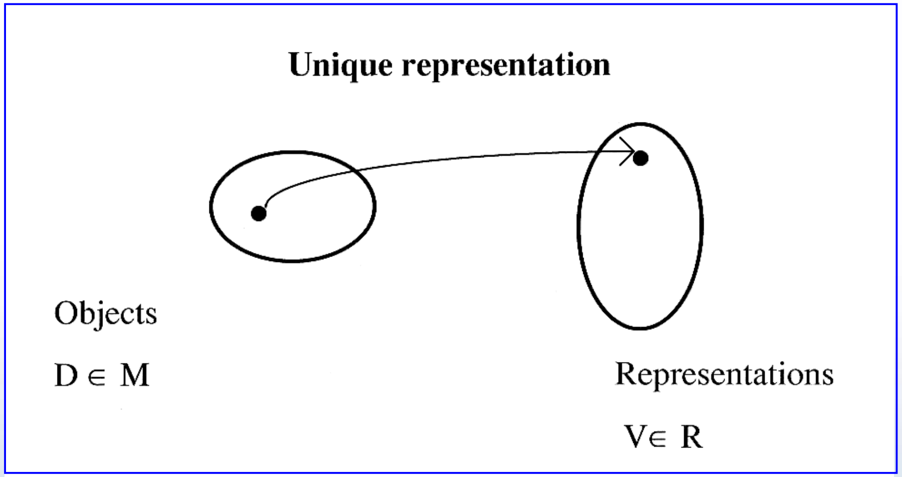
- Completeness (non-ambiguity)

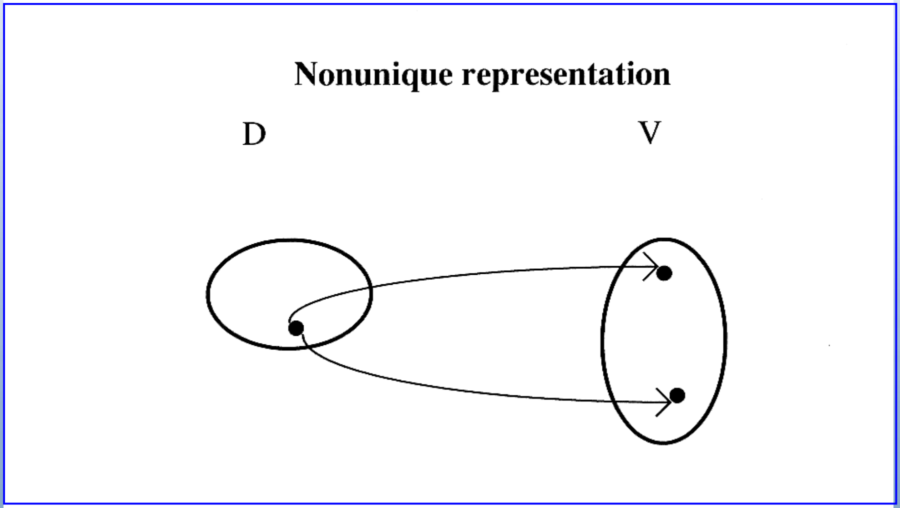
A representation $r \in V$ is unambiguous if it corresponds to a <u>*single*</u> *object in D*. A scheme is complete or unambiguous if all of its valid representations are unambiguous.

Formal properties of representations

- Uniqueness. A scheme is unique if for one object there is one and only one representation.

Representational schemes which are both unambiguous (complete) and unique are highly desirable because they are one-to-one mappings.





Informal properties of representation schemes

- Conciseness.

  This refers to the “size” of representations in a scheme (proportional to required memory). Compare: equation and polygonal model.

- Ease of creation.

  Concise representations generally are easier to create. Input subsystems are needed to help users to create representations.

Informal properties of representation schemes

- Suitability for applications.

Example: Roman numbers are not convenient for arithmetic operations. In solid modelling, no single representation is uniformly “best”.

Multiple representations are suitable for general-purpose solid modelling systems.

Ambiguous schemes

1) Engineering drawings (drafts).

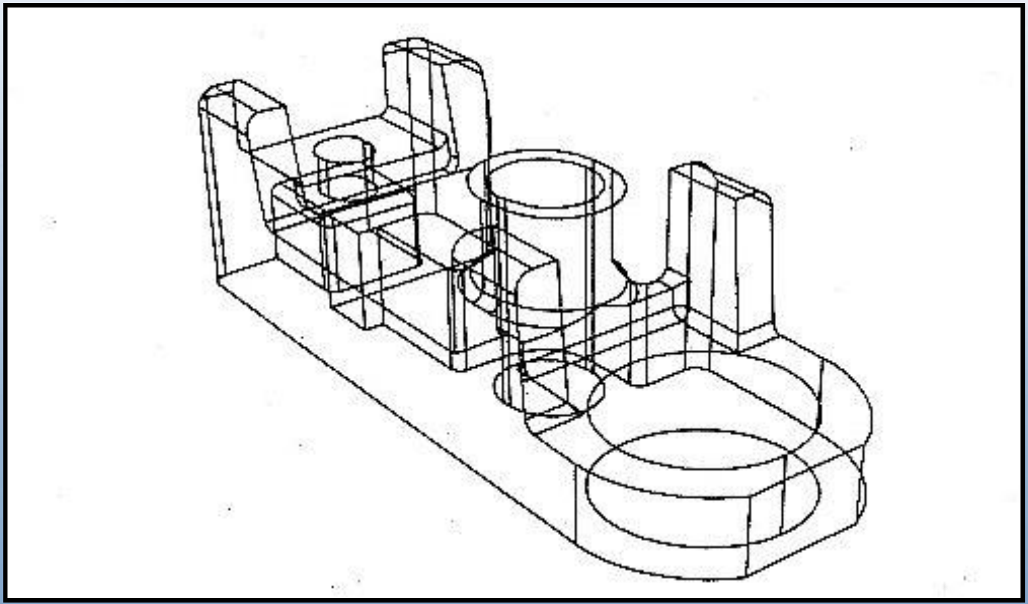
No formal definition as a representation scheme.

More formal corresponding schemes:

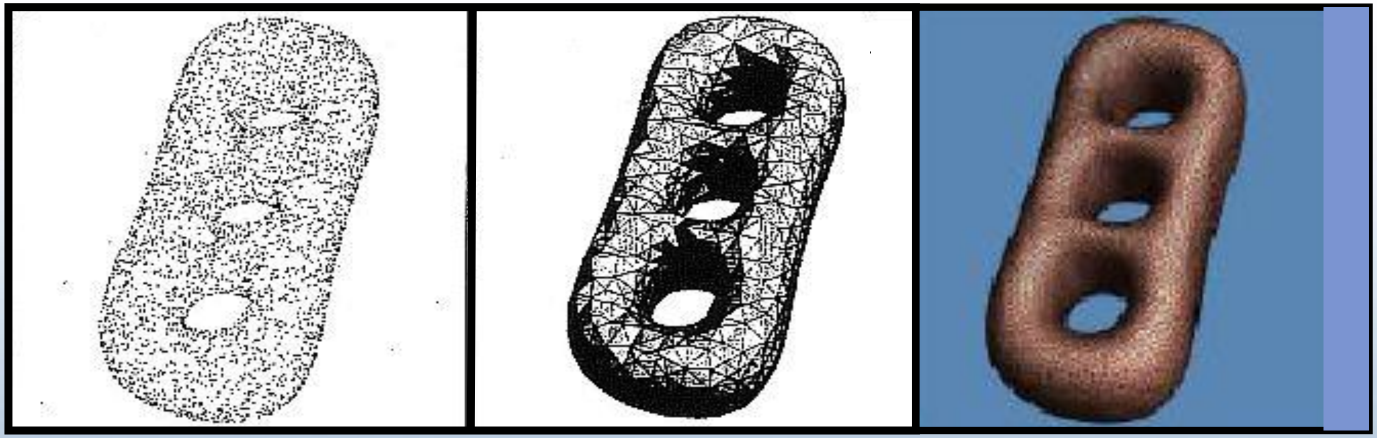
- Collections of planar projections.

  Mapping to a 3 solid is needed.

- Suitable collections of 3D entities. Selection of “edges” leads to wire frame representations.



2) Measurements of physical solids.



(a) Input: 4000 unorganized points P

(b) Step 1: Reconstructed mesh M

(c) Step 2: Smoothed mesh

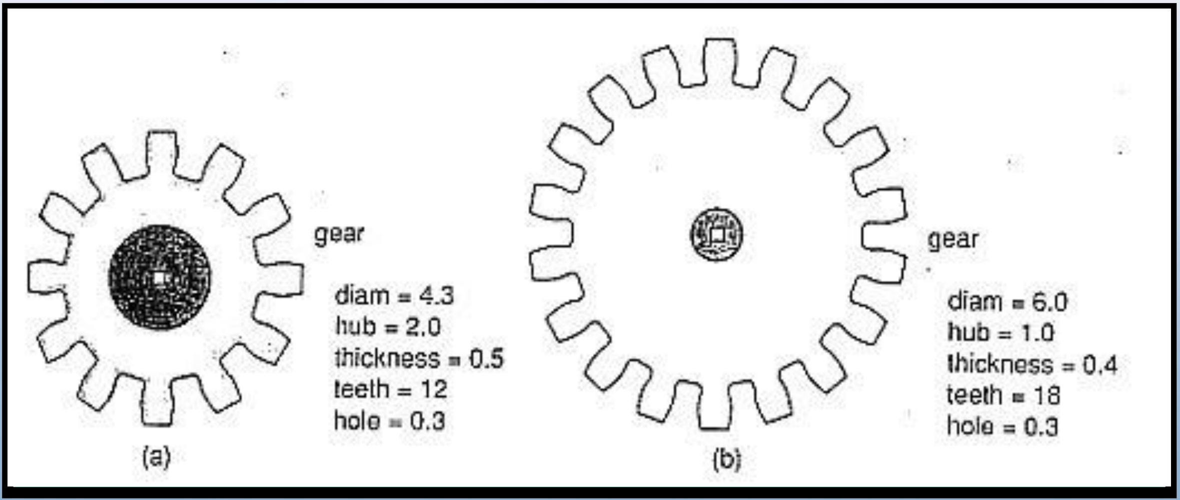
A solid is represented by a set of coordinates of points lying on the boundary or inside the object.

Pure Primitive Instancing

The modelling system defines a set of primitive 3D solid shapes for the specific application area:

$$\textbf{primitive}_i\,(\mathbf{a}_1,\mathbf{a}_2,\ldots,\mathbf{a}_k)$$

- Primitive with parameters define a family of parts;
- Primitives may include complex objects (gears, bolts, etc.);



Two gears defined by primitive instancing.

Pure Primitive Instancing

- No operations to form a new more complex object;
- Only one way to create a new kind of object - to write the code that defines it;
- Programs to draw or to calculate mass properties must be written individually for every primitive.

Pure primitive instancing

Properties:

- Unambiguous.

  A set of parameters defines one solid.

- Unique.

  For a solid only one set of parameters exists.

- Concise and easy to validate.

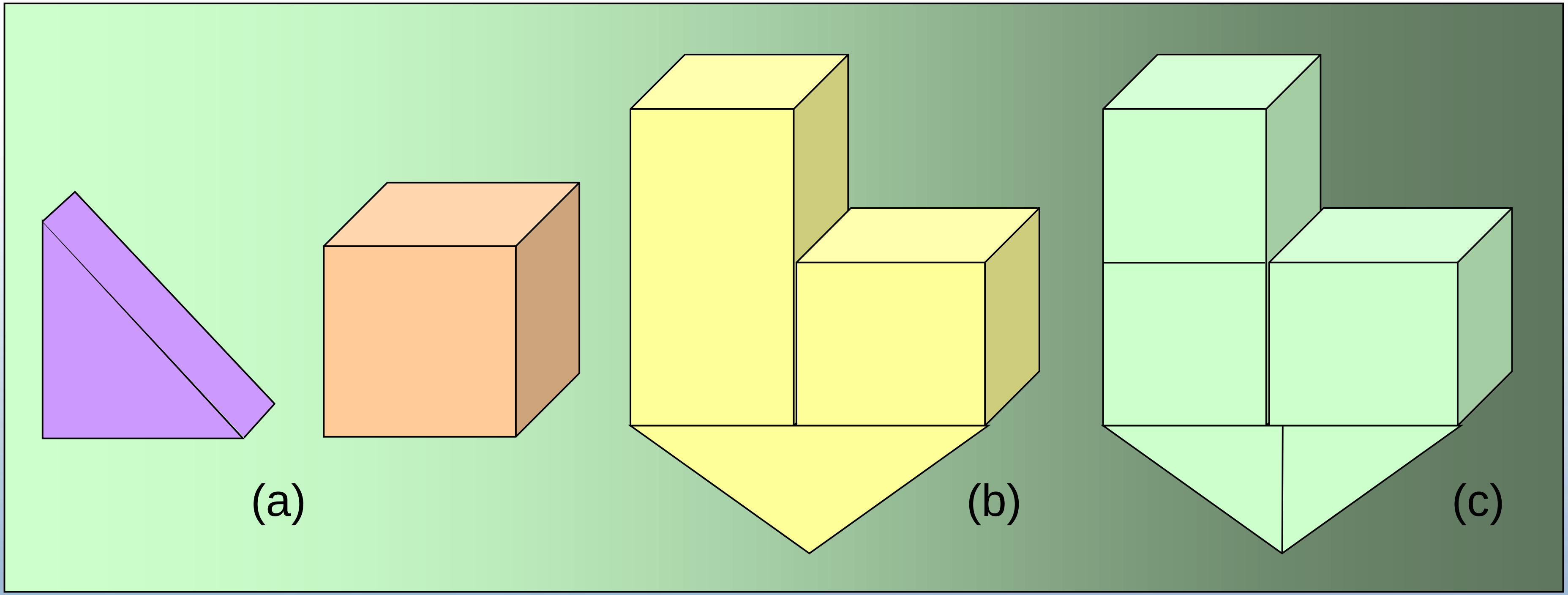
Properties

- Easy to use.
- Domains are small enough to be covered by a small catalog of primitives with small number of parameters.
- Efficient in specific applications but allow no uniform treatment.

Cell decompositions

- Parameterized set of primitive cells that are often curved
- Constructing complex object by “gluing” primitive cells together
- Restrictions on the “glue” operation often require that two cells share a single point, edge, or face

- Representation is not unique:



The cell shown in (a) may be transformed to construct the same object shown in (b) and (c) in different ways.

Properties

- Unambiguous

  A set of cells defines one solid

- Validity

  is computationally expensive to establish

- Not concise

- Not easy to create

Properties

- Convenient

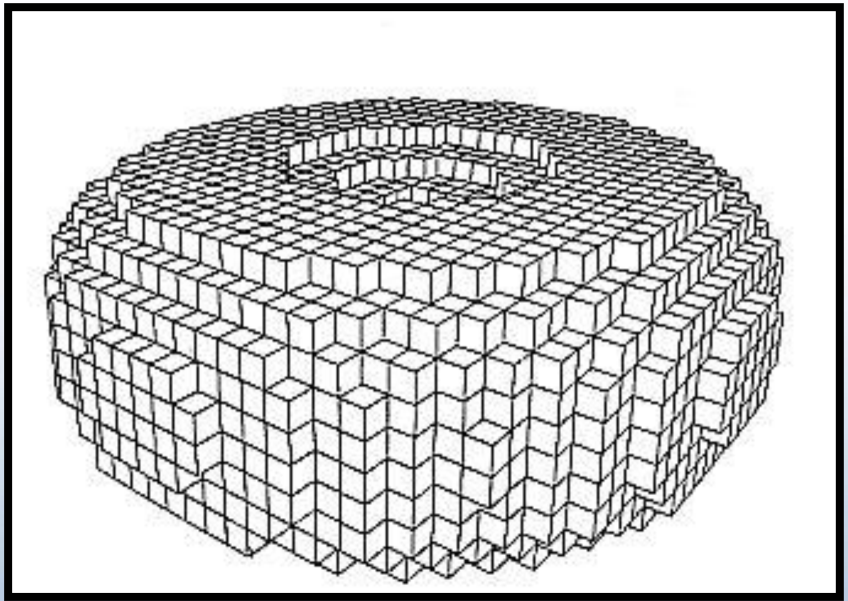
for computing topological properties:

- "one-piece" detection (connectivity)
- check "voids" or "holes"

Cell decomposition is used in 3D finite element methods (FEM) for the numerical solutions of differential equations.

Spatial occupancy enumeration

- Special case of cell decomposition with identical cells arranged in a fixed, regular grid
- The cells are often called voxels
- The most common cell type is the cube, and the representation of space as a regular array of cubes is called a voxel array (a cuberrile)



Torus represented by spatial-occupancy enumeration.

Spatial occupancy enumeration

- For every cell, only its presence or absence in the grid is defined
- A cell is presented in the grid if it is occupied by the object
- Disadvantages:

 - approximate model, no concept of “partial occupancy”

 - memory consuming (up to n3 cells)

Properties

- Unambiguous.

  A set of voxels defines one solid.

- Unique.

  For a solid only one set of voxels with the given grid step exists.

Properties

- Easy to validate.
- Not concise (verbose).
- Efficient in applications where objects are boxlike (architecture) or extremely irregular (biomedical).

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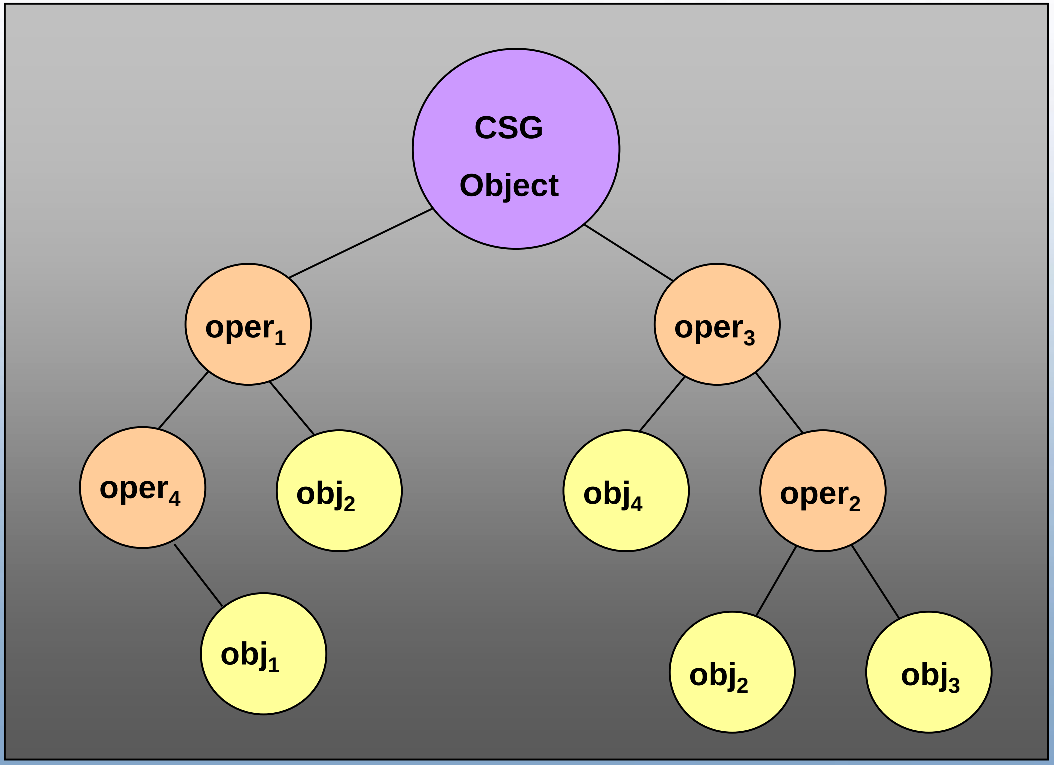
"Still with Bolts" by Jaime Vives Piqueres (2002)

Constructive Solid Geometry (CSG)

- CSG is based on a set of 3D solid primitives and regularized set-theoretic operations
- Traditional primitives: block, cylinder, cone, sphere, torus
- Operations; union, intersection, difference + translation and rotation

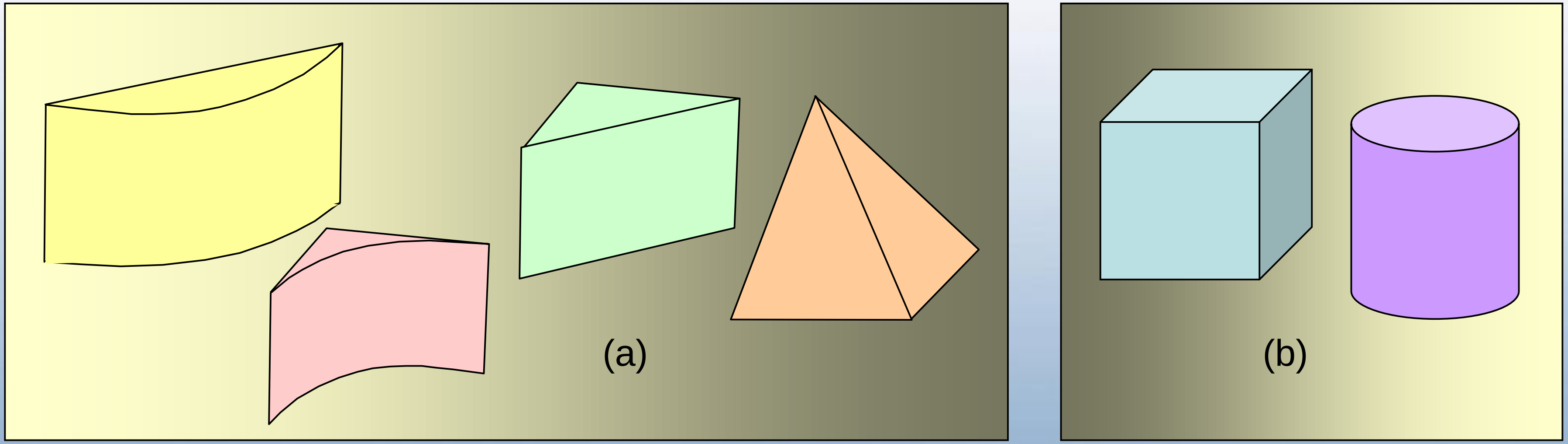
CSG tree

- A complex solid is represented with a binary tree usually called CSG tree



Properties

- Domain depends on the set of primitives and on the set of operations.



Two CSG schemes having different primitives but the same domain.

Properties

- Unambiguous

  A CSG tree defines one solid.

- Nonunique

  There are several possible CSG trees for one solid.

- Validity

  Any syntactically correct CSG tree is **valid**, if the primitives are r-sets.

  - CSG tree based on unbounded primitives may represent bounded sets and therefore be invalid.

- Concise

  if primitives are well matched to the domain

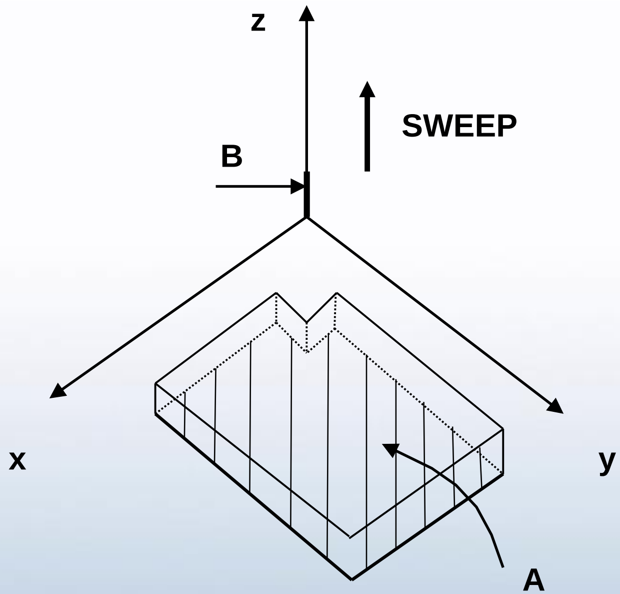
- Humans can easily create CSG representations

- Efficient

  for rendering and computing integral properties; not efficient for line drawings and certain types of graphic interactions (“pick an edge”).

Sweep representation

- A set of all points visited by an object A moving along a trajectory B is a new solid, called a sweep.
- Translational sweeping (extrusion): 2D area moves along a line normal to the plane of the area.



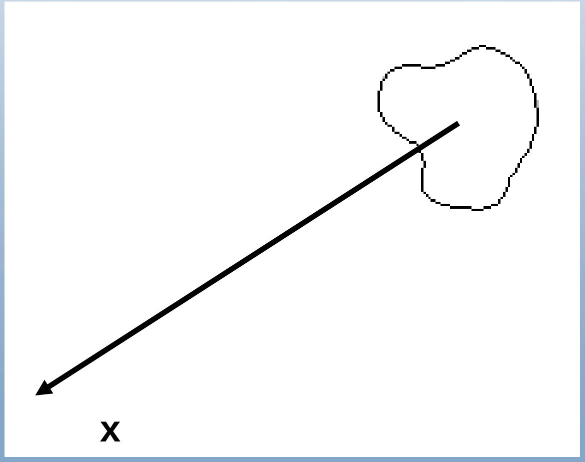
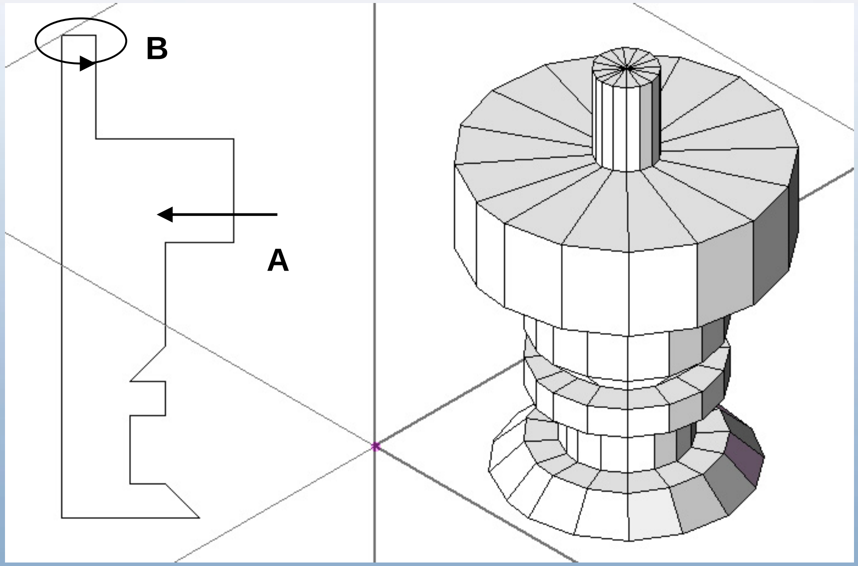


Image by Martin Culpepper, 1999

- Rotational sweeping is defined by rotating an area about an axis



www.tipus.uniroma3.it

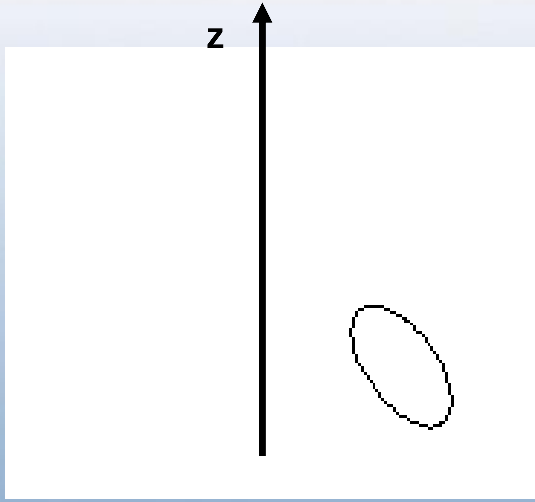


Image by Martin Culpepper, 1999

- Sweeps with a generating area changing in size, shape or orientation and following an arbitrary curved trajectory are called generalized cylinders.

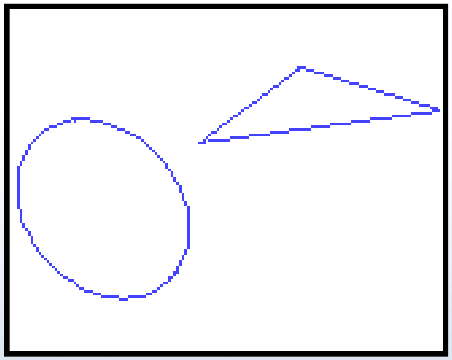


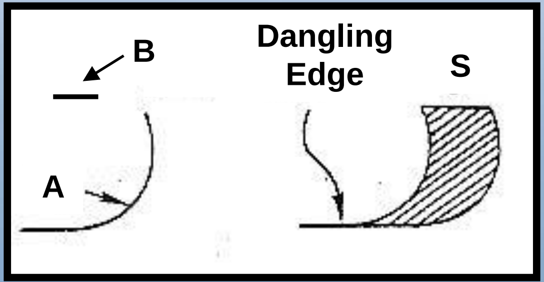
Image by Martin Culpepper

Problems: sweeping by moving solid, self-intersections, CSG operations on sweep.

Properties

- Unambiguous

  A moving object + a trajectory define one solid
- Not unique
- General validity conditions for sweep representations are unknown. General sweeping may produce non-regular sets.



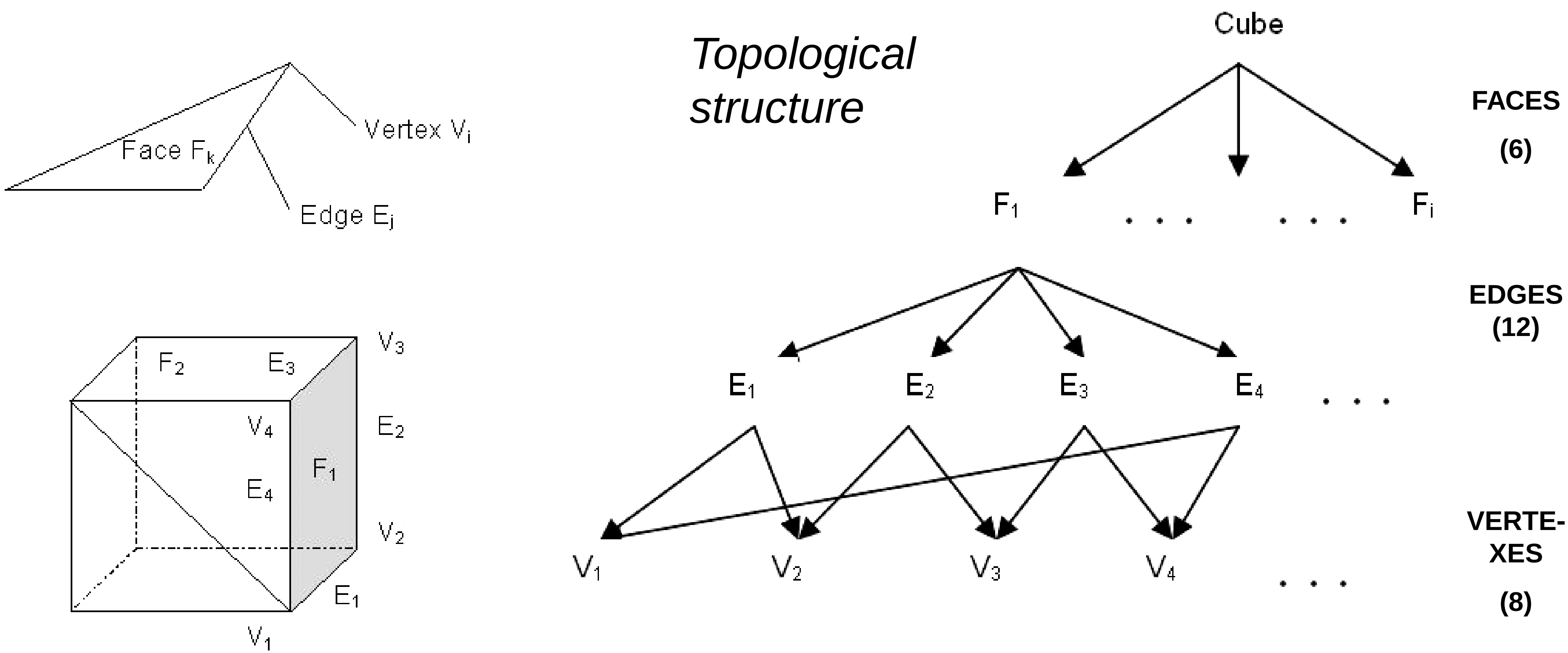
Requicha, Comp. Surveys, p.451

- Applications of sweeping by a moving solid: material removal (NC machining), dynamic interference of solids.

- Sweeping by a moving solid: lack of known algorithms for computing properties.

Boundary representation

Example: A boundary representation for a cube



Properties

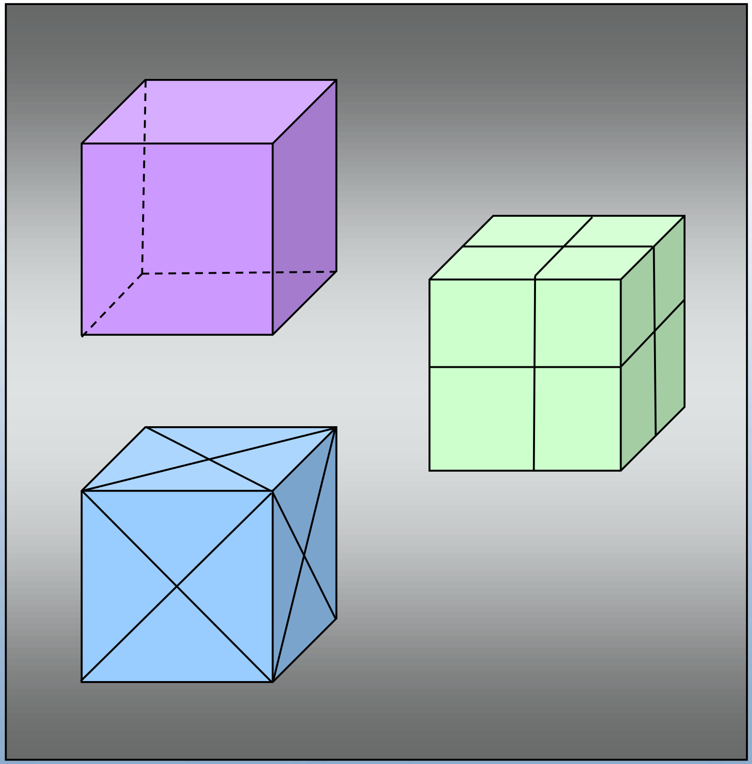
- Domains

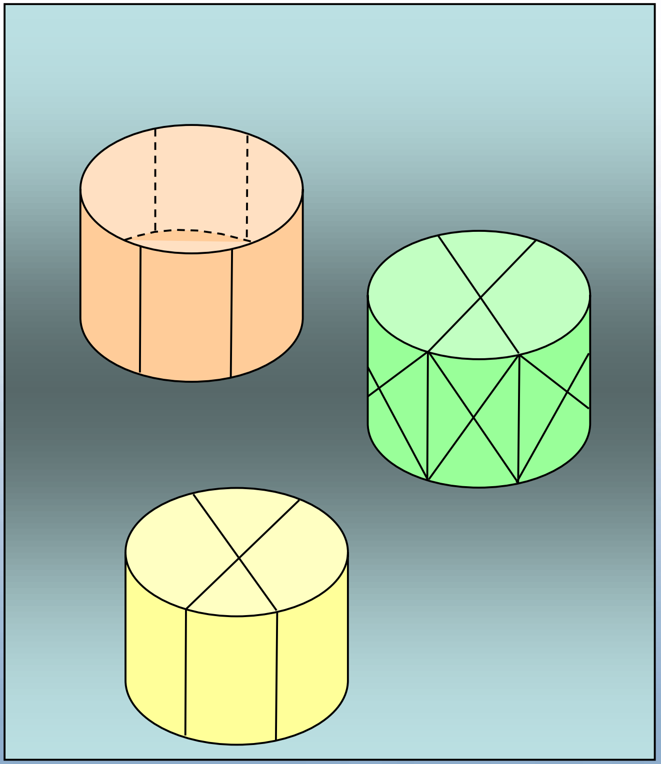
  are as reach as those of cell-decomposition or CSG schemes. Given a CSG scheme it is always possible to design BRep scheme with the same domain.

- Unambiguous

  if faces are represented unambiguously.

- Not unique





Properties

- Validity
  control requires expensive calculations.
- Not concise (verbose)
  More than 10 times longer than corresponding CSG.
- Difficult
  for humans to construct.
- Efficient
  in line and shaded drawings, graphic interaction and topological applications.

ARISTIDES A. G. REQUICHA *Representations for Rigid Solids*

SUMMARY OF REPRESENTATION SCHEME PROPERTIES AND APPLICABILITY

Schemes		PROPERTIES						APPLICATIONS											
		DOMAIN	VALIDITY	COMPLETENESS	UNIQUENESS	CONCISENESS	EASE OF CREATION	NULL OBJECT	OBJECT EQUALITY	LINE DRAWINGS	GRAPHIC INTERACTION	SHADED DRAWINGS	MASS PROPERTIES	HOMOLOGICAL PROPERTIES	FINITE ELEMENT ANALYSIS	INTERFERENCE ANALYSIS	ROUGH MACHINING	FINISH MACHINING	NC TAPE VERIFICATION
PRIMITIVE INSTANCING			G	G	G	G	G	G	G	G		?	G	G	?		G	G	G
SPATIAL ENUMERATION			G	G	G			G	G	?	?	?	G	G	G	G			
CELL DECOMPOSITION		G		G				G		?	?	?	P	G	P	?	?	?	?
CSG	GENERAL 1/2 SPACES	G		G				?	?		P	?	?			P	P	?	P
CSG	BOUNDED PRIMITIVES	G	G	G		G	G	?	?		P	?	?			P	P	?	P
SWEEP	TRANSLATIONAL & ROTATIONAL		G	G	?	G	G	G	?	G	G	?	G	?	P				
SWEEP	GENERAL	G		G				G											
BOUNDARY		G		G	?			G	?	G	G	P	?	P				P	

KEY: G = GOOD P = PROMISING ? = UNCLEAR BLANK= UNPROMISING OR POOR

H. Voelcker, 1995

A Taxonomy of Complete Representation Schemes for Solids

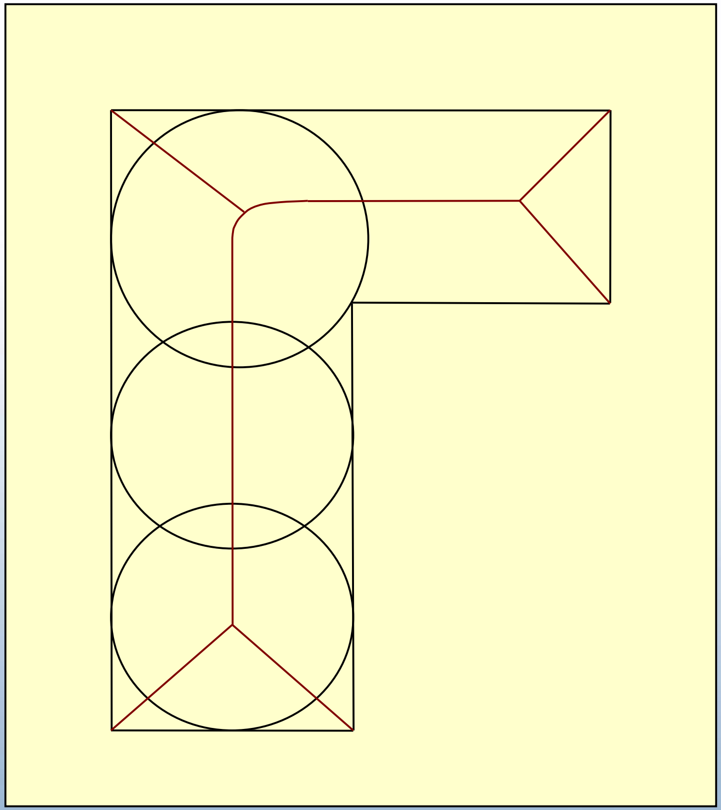
< H. Voelcker: 9/93; rev. 2/94, 2/95, 6/95 >

PARADIGM	FAMILY NAME	VARIANT	MATHEMATICAL MECHANISM	ACCESSIBLE ATTRIBUTES	SPECIAL PROPERTIES
Instantiation	Templates[1]		Special-Case Formulae		
Cell Composition (Gluing)	Spatial Enumeration[2]	Linear subdivision of space ⇒ rectilinear grids of equal-size cells	Union of Quasi-Disjoint Solid Cells $Solid = \cup C_i$ where $C_i \cap_n C_j = \varnothing, i \_ j$	Interior, Discretized into Cells	Spatial Addressability
		Recursive subdivision ⇒ hierarchically graded cells (quadtrees, octrees)			
	Cell Decomposition	Simplicial or Cell Complexes; elements produced by object partitioning (e.g. triangulation)			
	Boundary Representations (B-reps)	Exact boundaries	Union of Boundary Cells $\partial Solid = \cup Face_i$ $\partial Face = \cup Edge_i$ (Solids thru boundary determinism)	Boundary; Surface 'Features'	
		Linear approximations ('tilings', 'facets')			
Boolean Composition	Constructive Solid Geometry (CSG)	Halfspace primitives	Regularized boolean composition	Solid primitives; syntactic structure	Recursive partitioning
		Bounded solid primitives			
Generation	Sweep Representations	Simple sweeps: S constant, M a single rotation or translation	'Infinite Union': $Sweep(S, M(t)) =$ $\cup S(t_i) @ M(t_i), \ \forall t_i \in [interval]$	Generator Elements (S, M)	(Limited domains)
		General sweeps			
	Medial Axis Transforms (MATs)[3]		$\cup Spine(p) \oplus Sphere(R(p))$	Spine (skeleton); Radius function	
Spatial Sampling	Ray Representations[3]		Induced cell decompositions	Ray-sampled interior	Directional sampling
	Parametric Sample Sets[3]		Interpolation	Sampled boundary	
Real and Vector Functions	Parametric-function Representations[3]		Homeomorphic mapping over finite domains	Interior	(Limited domains)
	Real-function Representations[3]	Differentiable Indicator Functions	$f(p) > 0 \Rightarrow p \in iS$ $f(p) = 0 \Rightarrow p \in \partial S$ $f(p) < 0 \Rightarrow p \in cS$	Point classifications	Boolean composition through arithmetic operations

[1] Called 'Pure Primitive Instancing' in Requicha's paper in *ACM Computing Surveys,* December 1980.
[2] Called 'Spatial Occupancy Enumeration', without linear and hierarchical distinctions, in Requicha's 1980 paper.
[3] Not acknowledged in Requicha's 1980 paper.

Medial axis representations

The medial axis of a 2D object is defined as the closure of the locus of centers of maximal inscribed disks. A disk is maximal if no other disk contains it.

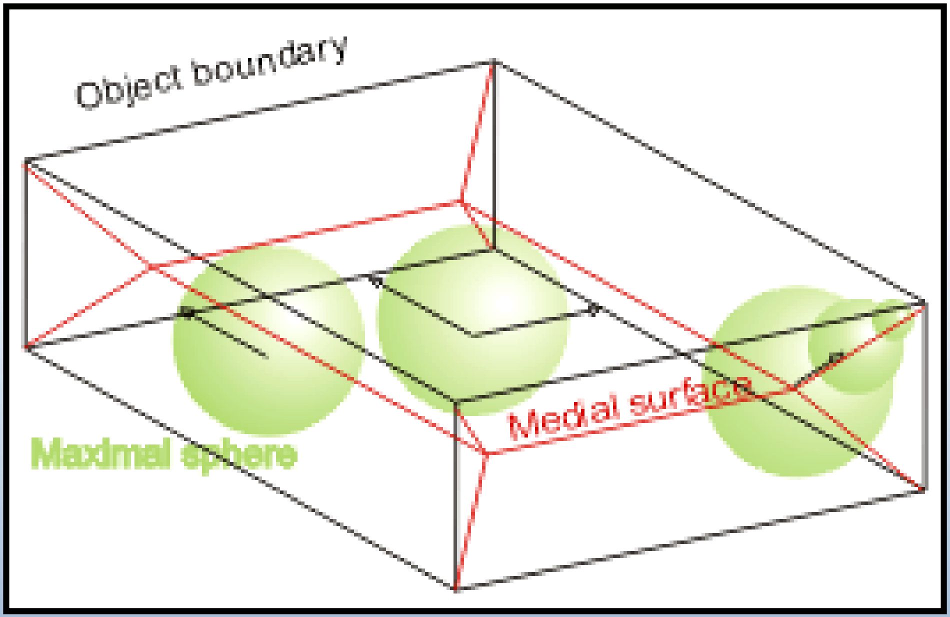


Hoffmann “Solid modelling” Overview

L-shaped domain and associated medial axis. Some maximal inscribed circles contributing to the medial axes are also shown.

Medial axis representations

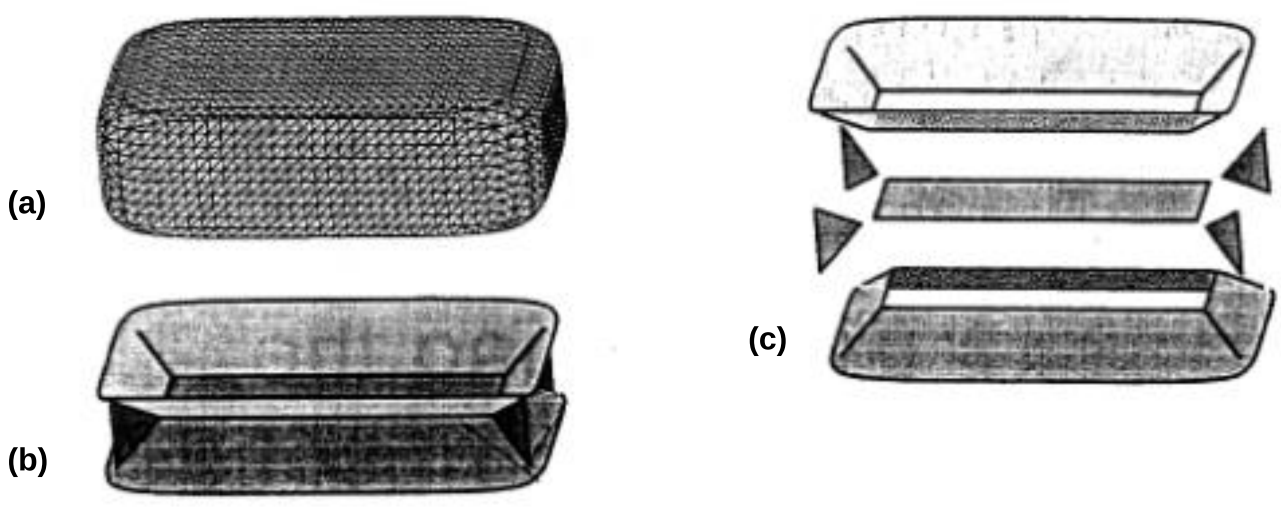
The medial surface of a 3D solid is the closure of the locus of centers of maximal inscribed spheres.



Finite Element Modelling Group, Queen's University, Belfast

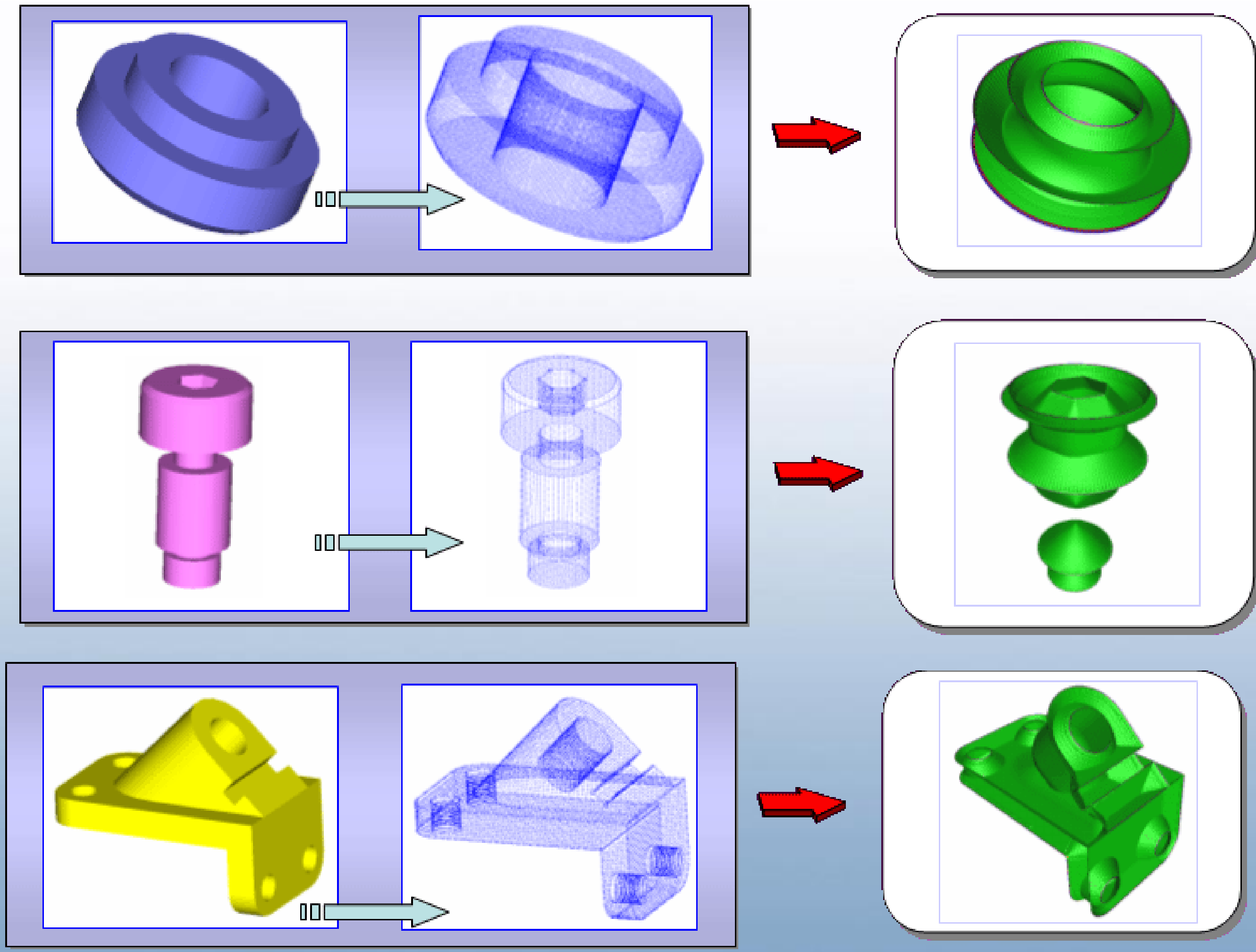
Morphology of 3D skeleton

(a) super-ellipsoidal block;
(b) skeleton of block with various element labeled;
(c) exploded skeleton showing separated patches.



Images by G.Turkiyyah et al.

Medial axis representations



Properties:

- Unambiguous
- Unique
- Efficient

in meshing algorithms

Images by T. K. Dey, H. Woo and W. Zhao

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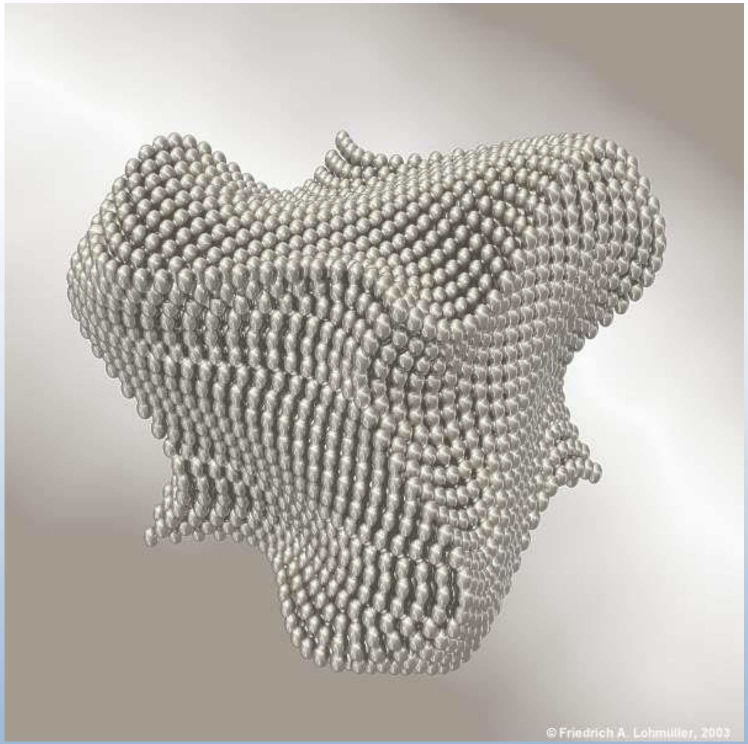
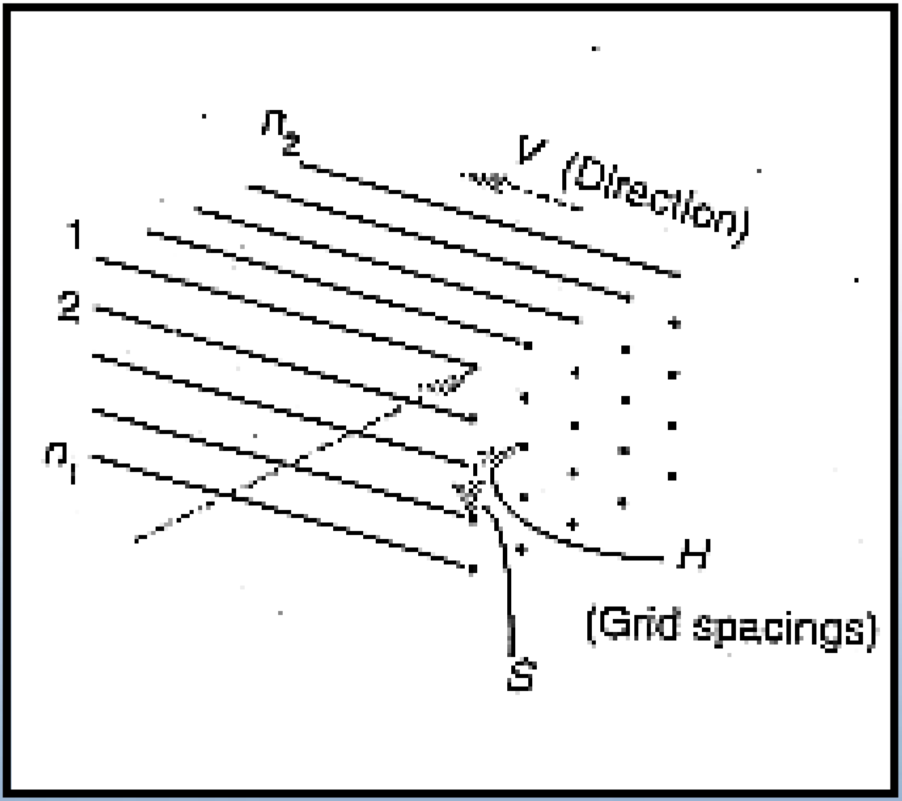


"Dancing Cube" by Friedrich A. Lohmueller (2003)

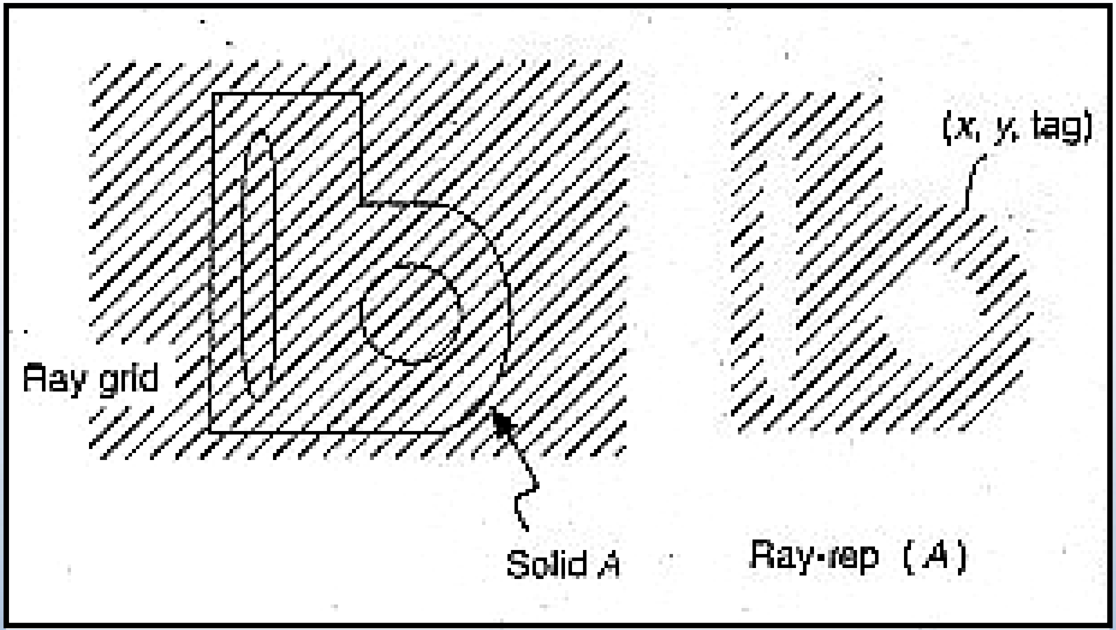
Ray representations

A ray grid is a finite set of regularly spaced parallel lines with an associated direction V.



J. Menon et al. CG&A, vol.14, N3, 1994

A ray representation of a solid is a set of segments resulting from the intersection of the ray grid with the solid.



A ray-rep might also contain tags, descriptive symbolic information appended to the segments. Tags can identify the primitive half-spaces in solid's CSG or faces in its B-rep.

Properties:

- Unambiguous

  under suitable conditions and with appropriate tags. Can be converted exactly from and to CSG and B-rep

- Not unique

  Depends on grid spacing

Parametric function representations

Shapes are represented by multidimensional, continuous, piecewise-differentiable parametric functions:

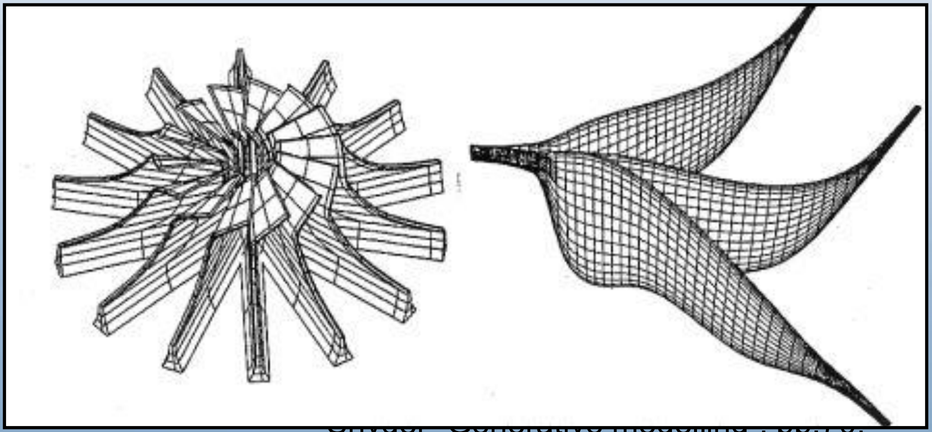
$$F: R^n \rightarrow R^m$$

where R^n is parameter space and R^m is object space.

For n=2, m=3

[x(u,v), y(u,v), z(u,v)]

defines a surface in 3D space.



Snyder "Generative modelling", pp.70,

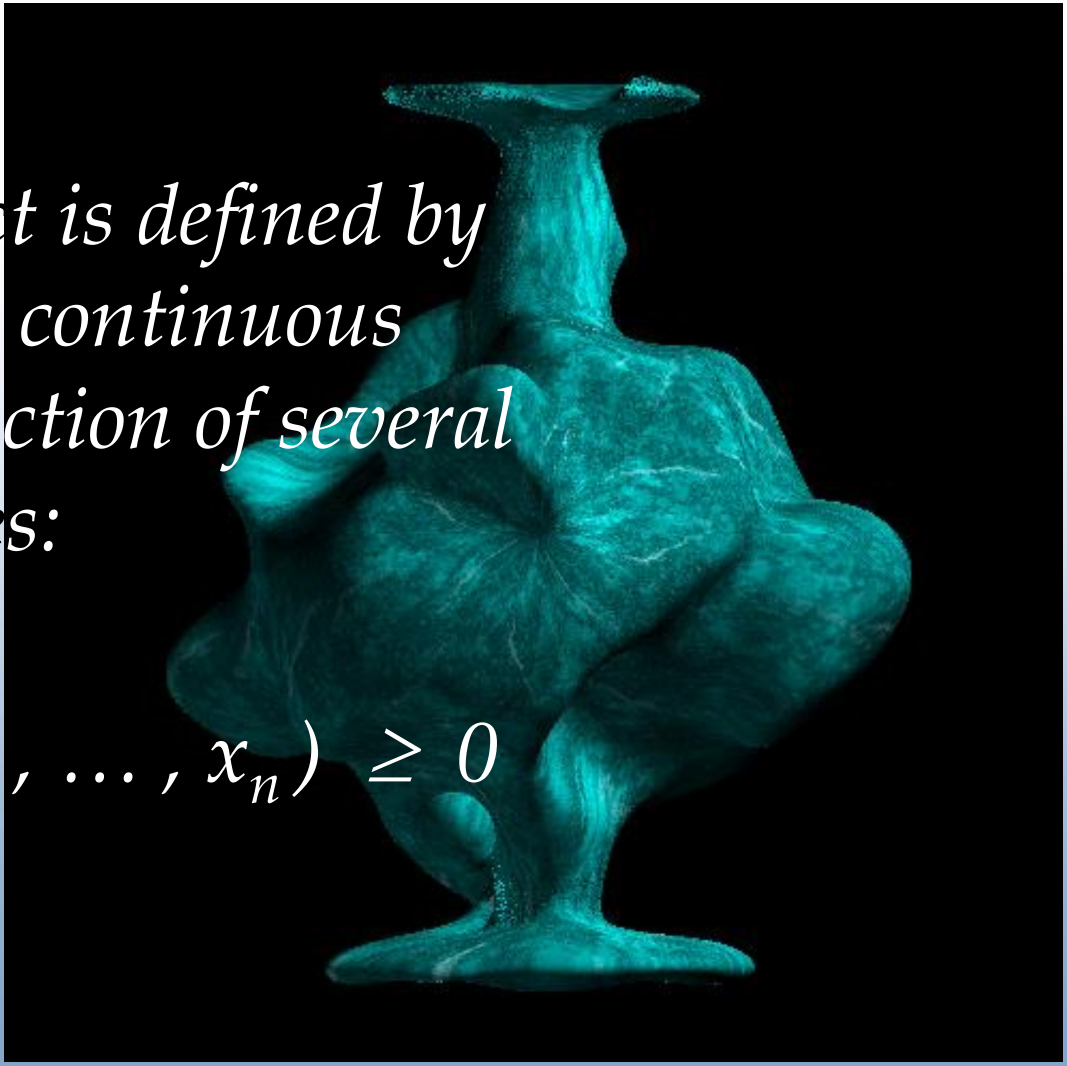
95

Properties

- Extension and generalization of sweeping
- Unambiguous, non unique representation
- Compact and easy to create
- Efficient algorithms

  with the use of interval analysis

Real function representations



An object is defined by a single continuous real function of several variables:

$$f(x_1, x_2, \ldots, x_n) \geq 0$$

- A function *f* can be defined analytically, with an evaluation algorithm, or with sampled values and an appropriate interpolation procedure.

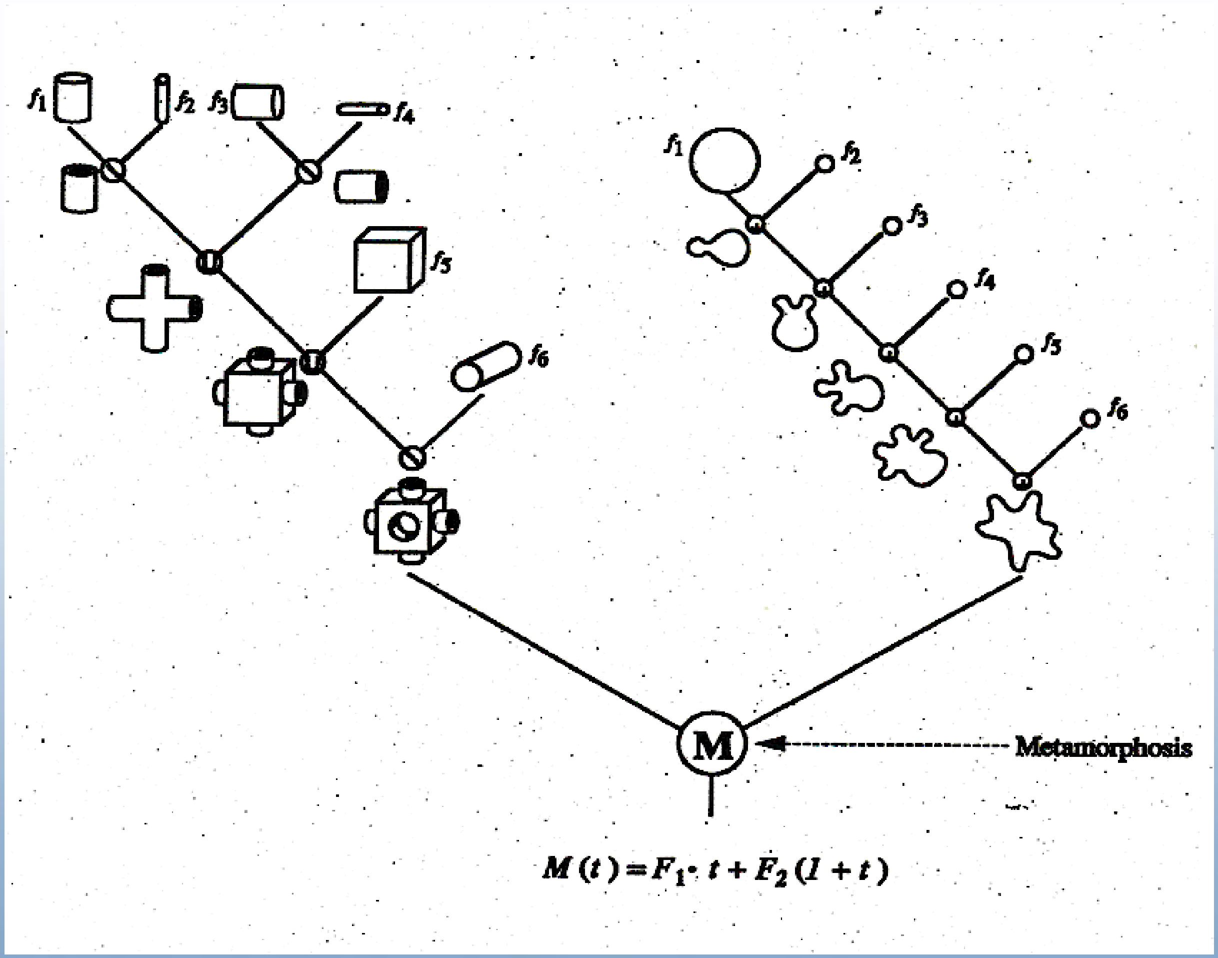
Function representation FRep

- Uniform representation of multidimensional objects defined as

$$F(X) \geq 0$$

- Function $F(X)$ evaluation procedure traversing the construction tree structure
- Leaves: primitives
- Nodes: operations + relations
- System extensibility

Construction of metamorphosis



Properties

- Closed under the arithmetic, set-theoretic, Cartesian product, projection and other operations.
- The abstraction level is higher than that of other known representations.

  Combinations of the following modelling styles are supported: CSG, sweeping, implicit and volumetric objects.

Real function representations
Properties

- Unambiguous

  A function defines one object

- Not unique

  for example, $k \cdot f \geq 0$

- Concise and easy to create

- Efficient

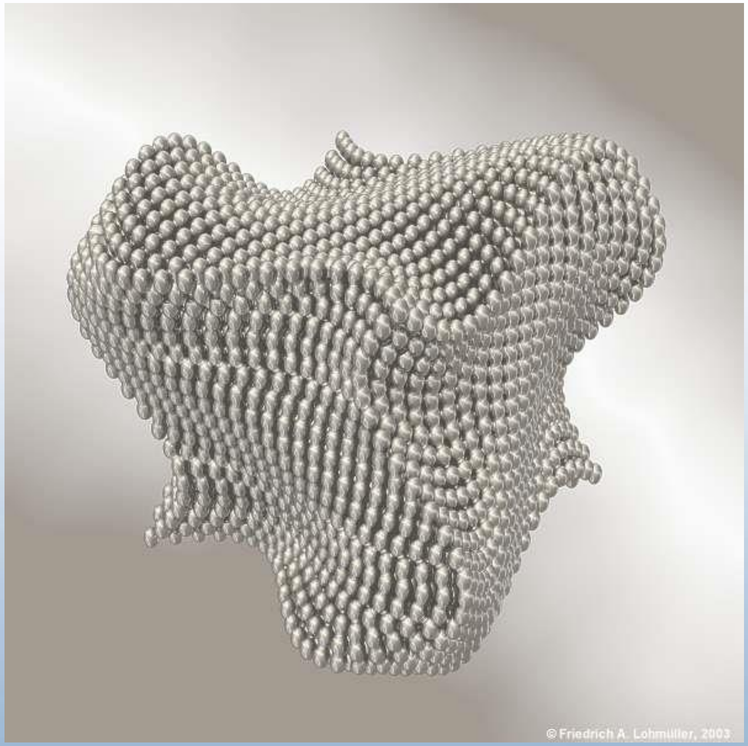
  in modelling highly complex objects

Contents

- Ray representations
- Parametric function representations
- Real function representations
- Hybrid schemes and conversions

3. Solid modelling systems

4. References



"Dancing Cube" by Friedrich A. Lohmueller (2003)

Hybrid schemes and conversions

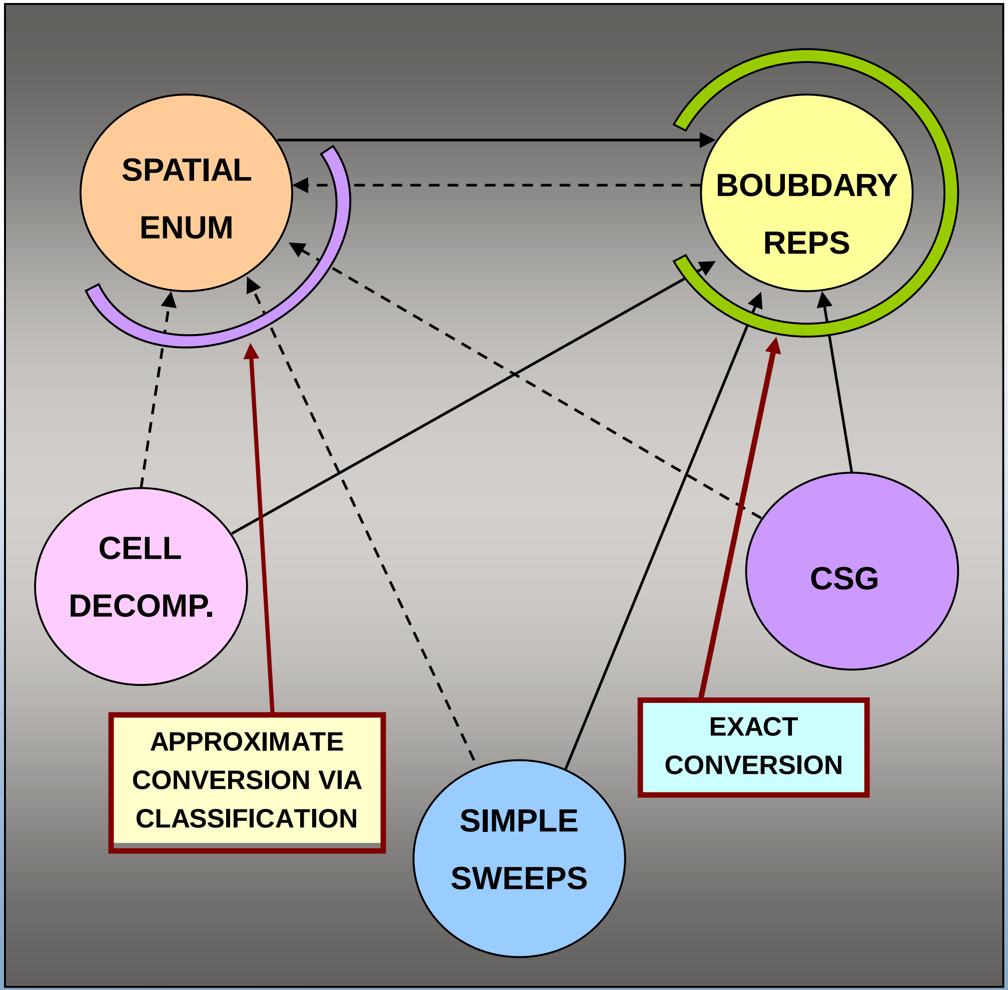
- ***Hybrid***, or ***non-homogeneous***, representation schemes may be designed by combining several schemes:

- 1) CSG/ boundary hybrid: CSG-like trees whose leaves are primitive solids or B-rep non-primitive solids.

  Is used as the basis for the input language of some systems.

Hybrid schemes and conversions

- 2) CSG/sweep hybrid: CSG-like trees whose leaves may be solid-sweep representations. Useful in numerically controlled (NC) machining and computer vision.



Requicha, Comp. Surveys, p.456

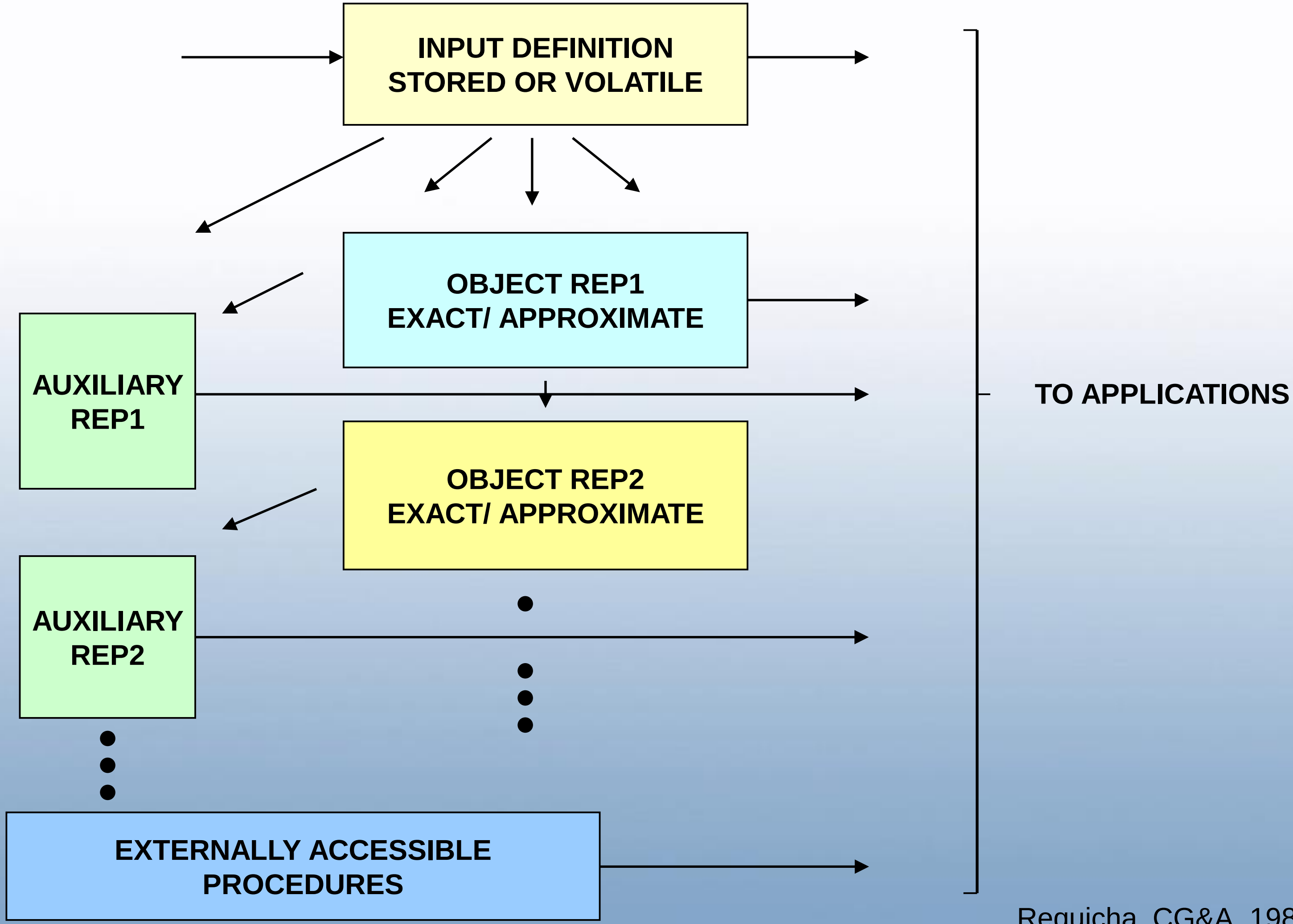
- Reasons for the lack of bidirectional exact conversions:

1) schemes such as sweeps have smaller domain than CSG, B-rep, or cell decompositions;

2) algorithms are not known.

- Exact conversion from CSG to B-rep ("*boundary evaluation*") requires nontrivial algorithms.

Architectures of solid modelling systems

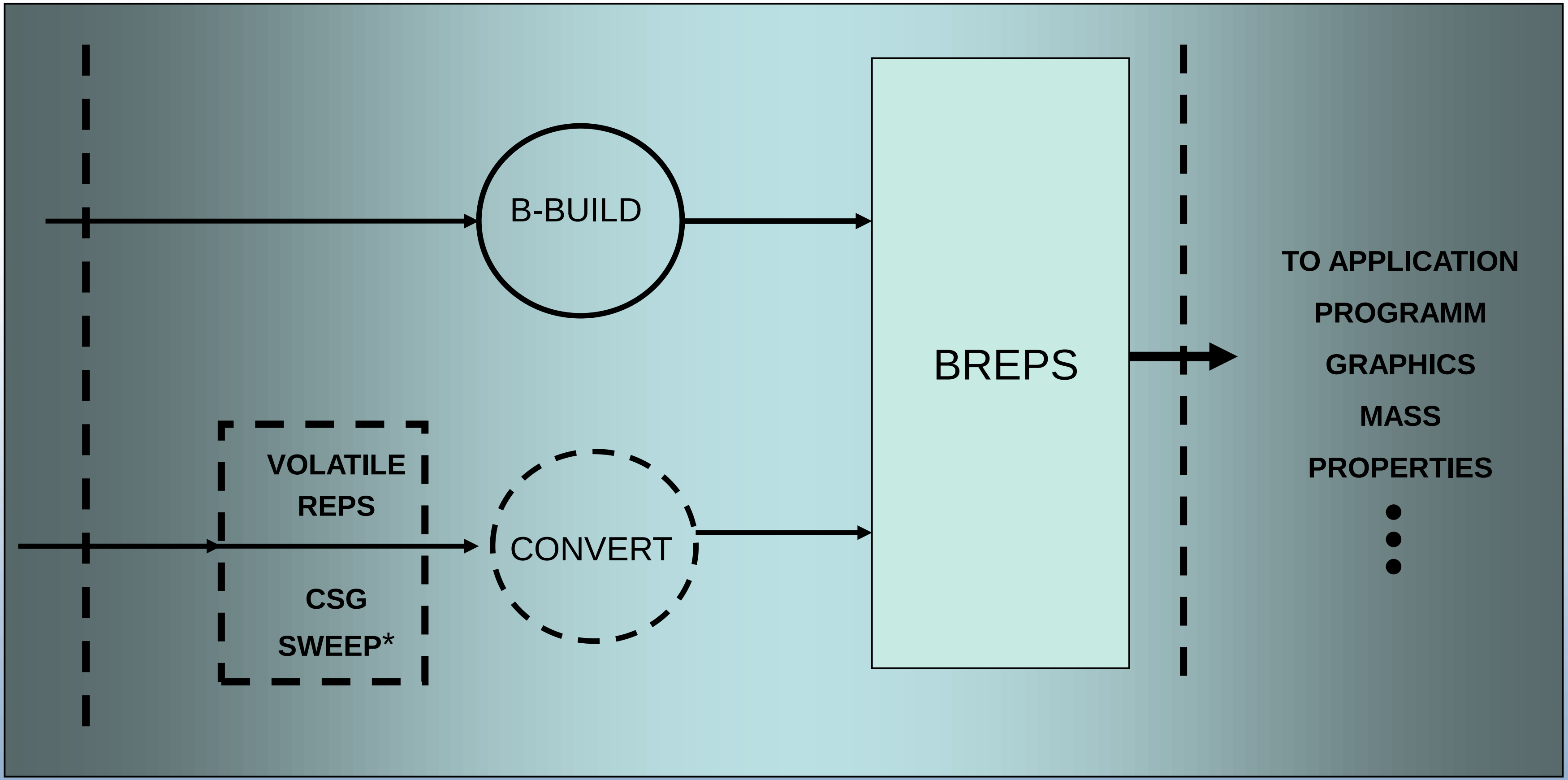


Requicha, CG&A, 1983, p.28

Architectures of solid modelling system

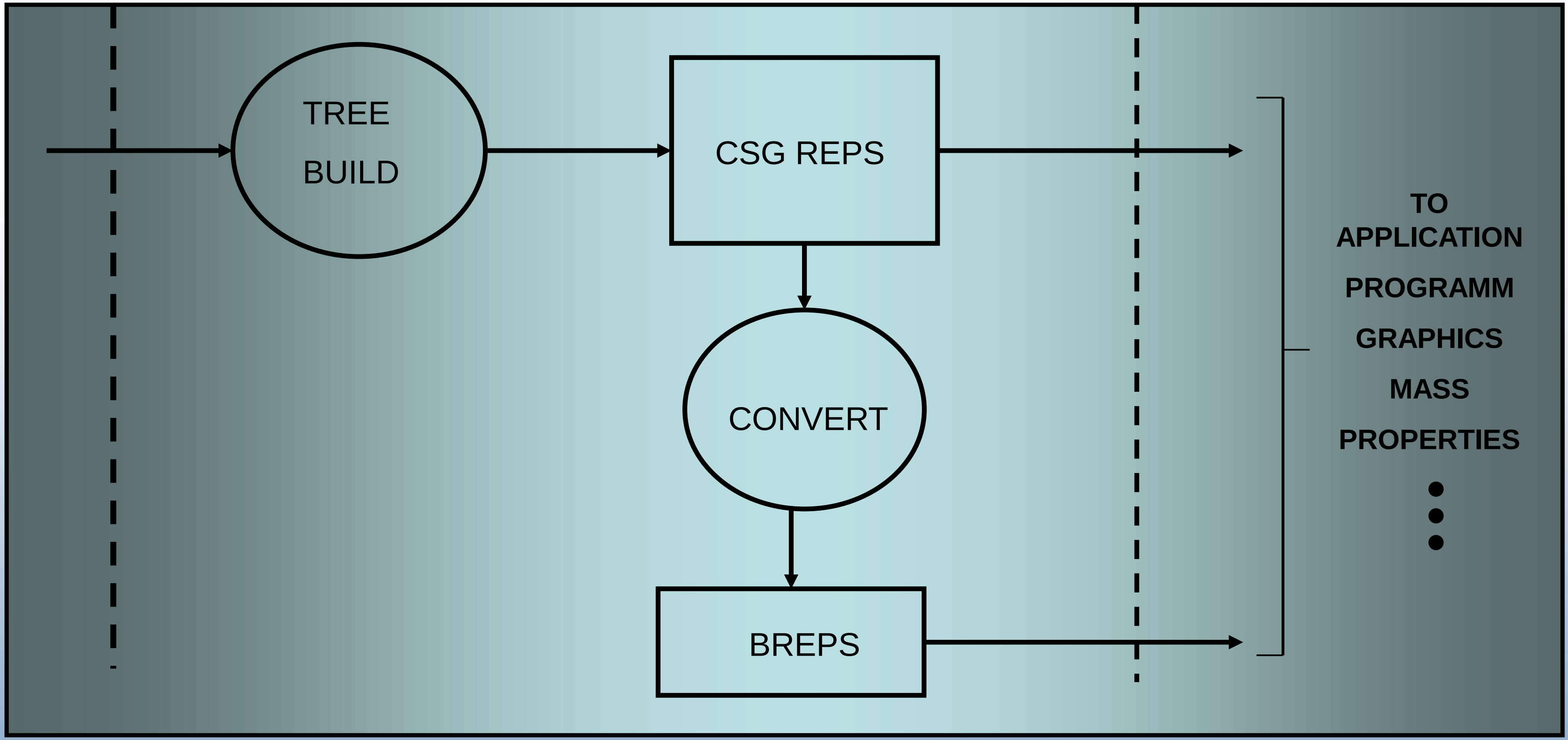
- A solid modelling system is regarded as
- 1) a specific collection of representations (exact, approximate, auxiliary) with at least one being valid and complete;
- 2) a collection of procedures for managing representations, conversions, and other geometrical calculations.
- Applications: graphics, mass properties calculations, finite-element meshing, interference checking and path planning, mechanism simulation, rapid prototyping, manufacturing, data storage and exchange.

Single representation and hybrid systems



Single (boundary) representation systems

Single representation and hybrid systems



Hybrid CSG/ BRep systems

Representational facilities of selected geometric modeling systems.

SYSTEM	PRIMARY REP SCHEME	DOMAIN (DEFINED BY HALFSPACES)	OTHER CONSISTENT REPS	INPUT BASED ON	INPUT MODALITIES
SHAPES	CSG:	QUAD	—	CSG	SC;T/I;IG*
TIPS	HALFSPACES	"ARBITRARY"	—	CSG	T/B
GDP/GRIN	CSG:	QUAD	≃B-REP	CSG	SC;IG
PADL-1	BOUNDED	PL & CYL (⊥)	B-REP	CSG	T/I
PADL-2	PRIMITIVES	QUAD	B-REP	CSG	SC;T/I
SYNTHAVISION		QUAD, SS	WIREFRAME	CSG	T/B
GMSOLID	HYBRID CSG/B-REP	QUAD	B-REP	CSG, SWEEP*	IG
U.M./BORKIN	B-REP	PL	—	CSG, SWEEP*	T/I;IG*
BUILD-2	B-REP	QUAD	—	CSG, EOP	SC;T/I
CADD	B-REP	QUAD, SS	—	SWEEP*	IG
COMPAC	B-REP	QUAD	—	CSG, SWEEP*	T/B?
DESIGN	B-REP	QUAD	—	CSG	IG
EUCLID	B-REP	≃QUAD	—	CSG, SWEEP*	T/I;IG?
GLIDE	B-REP	PL	—	CSG, SWEEP*, EOP	SC;T/I
MEDUSA	B-REP	≃QUAD, SS	—	CSG, SWEEP*	IG
PROREN-2	B-REP	QUAD	—	CSG, SWEEP*	T/I
ROMULUS	B-REP	QUAD	—	CSG, SWEEP*, EOP	SC;T/I

PL = PLANE
QUAD = QUADRIC SURFACES (NOTE 1)
SS = SCULPTURED SURFACES
⊥ = ORTHOGONAL POSITIONING
≃ = APPROXIMATE
SWEEP* = TRANSLATIONAL AND/OR ROTATIONAL SWEEP

EOP = EULER OPERATIONS
SC = SUBROUTINE CALL
T/I = TEXT/INTERACTIVE ORIENTATION
T/B = TEXT/BATCH ORIENTATION
IG = SOPHISTICATED INTERACTIVE GRAPHICS
IG* = SIMPLE INTERACTIVE GRAPHICS

Requicha and Voelcker 1982

More late and modern systems

Kernel modelers

- Parasolid (EDS Unigraphics, Cambridge, UK)

  BRep solid modeler supporting free-form surfaces (development of Romulus)

  http://www.eds.com/

- ACIS (Spatial Technology, USA)

  B-rep object-oriented toolkit (development of BUILD)

  http://www.spatial.com/

- Designbase (Ricoh, Japan)

  B-rep solid modelling library with free-form surfaces

  www.ricoh.co.jp/designbase/

- SVLIS (Information Geometers, UK)

  CSG object-oriented kernel modeller

  http://www.bath.ac.uk/~ensab/G_mod/Svlis/svlis.html

B-rep modelers

- CATIA **(Dassault, France / IBM)**
  http://www.catia.ibm.com/

- AutoCAD **Release (Autodesk, USA)**
  based on ACIS
  http://www.autodesk.com/

- SolidWorks **(SolidWorks, USA)**
  PC+Windows interactive system based on Parasolid
  http://www.solidworks.com/

CSG based hybrid systems

- Ray Casting Engine RCE (Duke/ Cornell Universities, USA) VLSI parallel special-purpose computer based on CSG/ ray-rep

  http://www.cs.duke.edu/~kedem/RCE/RCE.html

- POVRay CSG/B-rep hybrid modeller for photorealistic rendering
  http://www.povray.org

- BRL-CAD (USA Army) CSG/ BRep solid modelling system

  http://www.brl-cad.org

FRep based systems

- HyperFun (Aizu-Hosei, Japan; Bournemouth, UK) special-purpose high-level language and tools
  http://www.hyperfun.org
- Symvol for Rhino (Uformia, Norway) FRep plug-in to Rhinocerous CAD system
  http://uformia.com/products/symvol-for-rhino/

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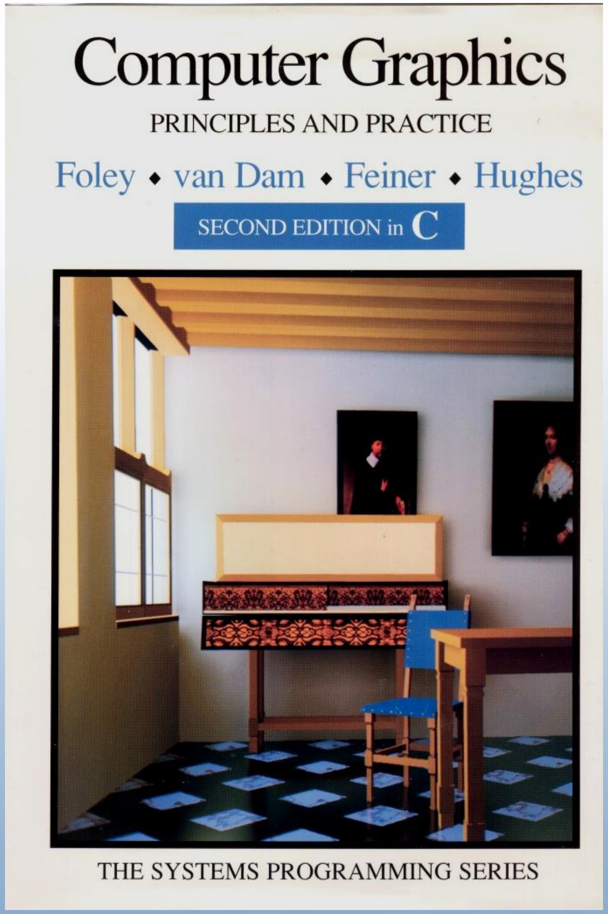

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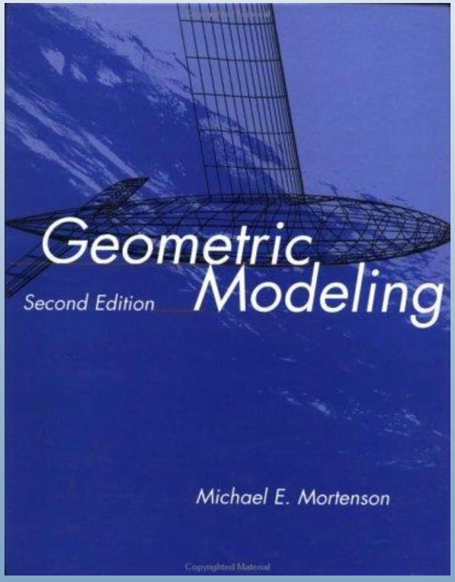

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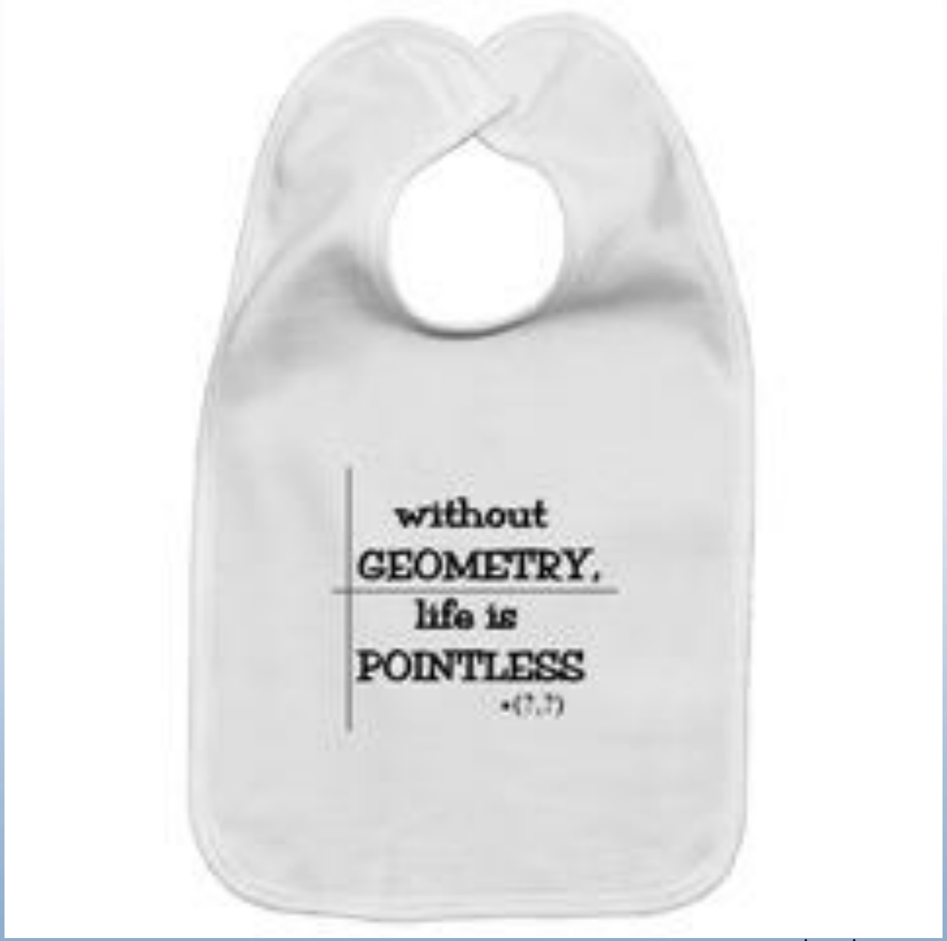
Words of wisdom

"…we are geometricians only by chance"

Dr. Johnson

"Without geometry, life is pointless"



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