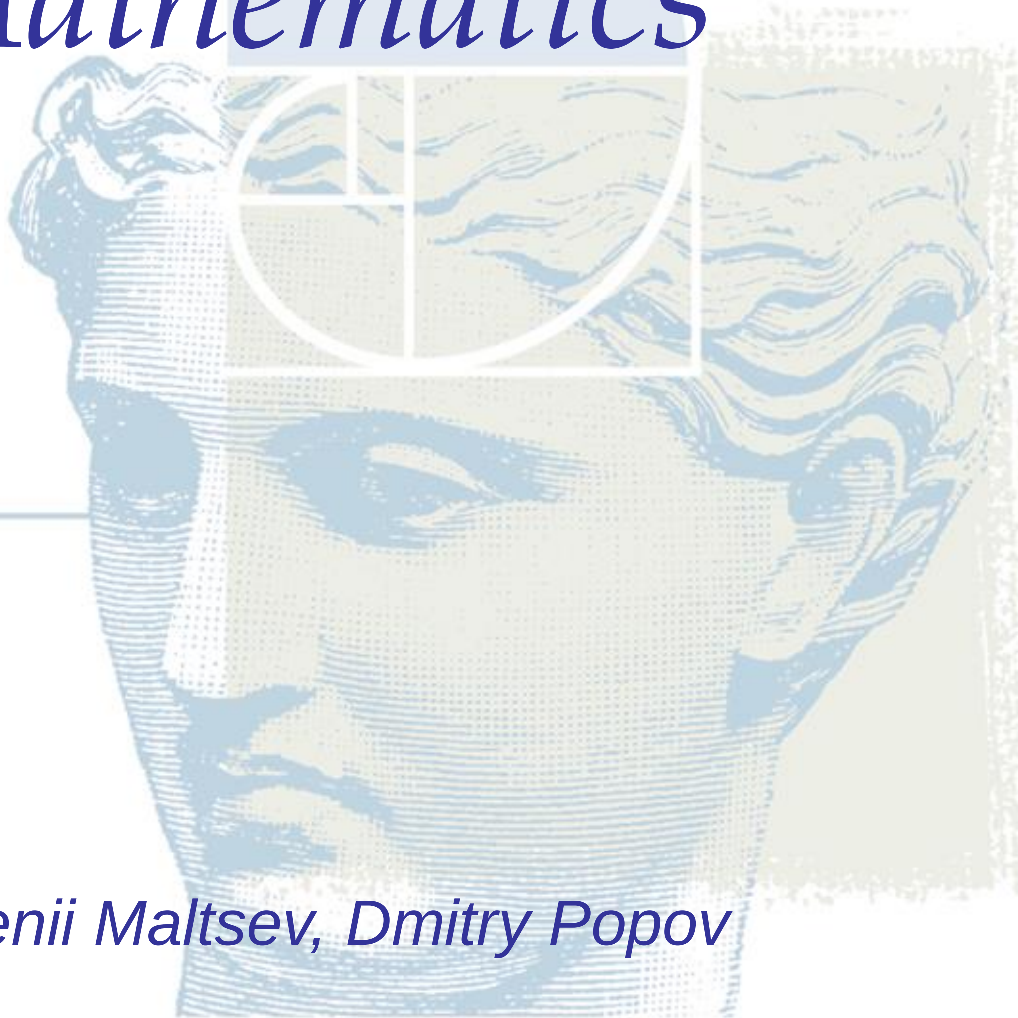
Discrete Mathematics



Alexander Pasko, Evgenii Maltsev, Dmitry Popov

Unit materials

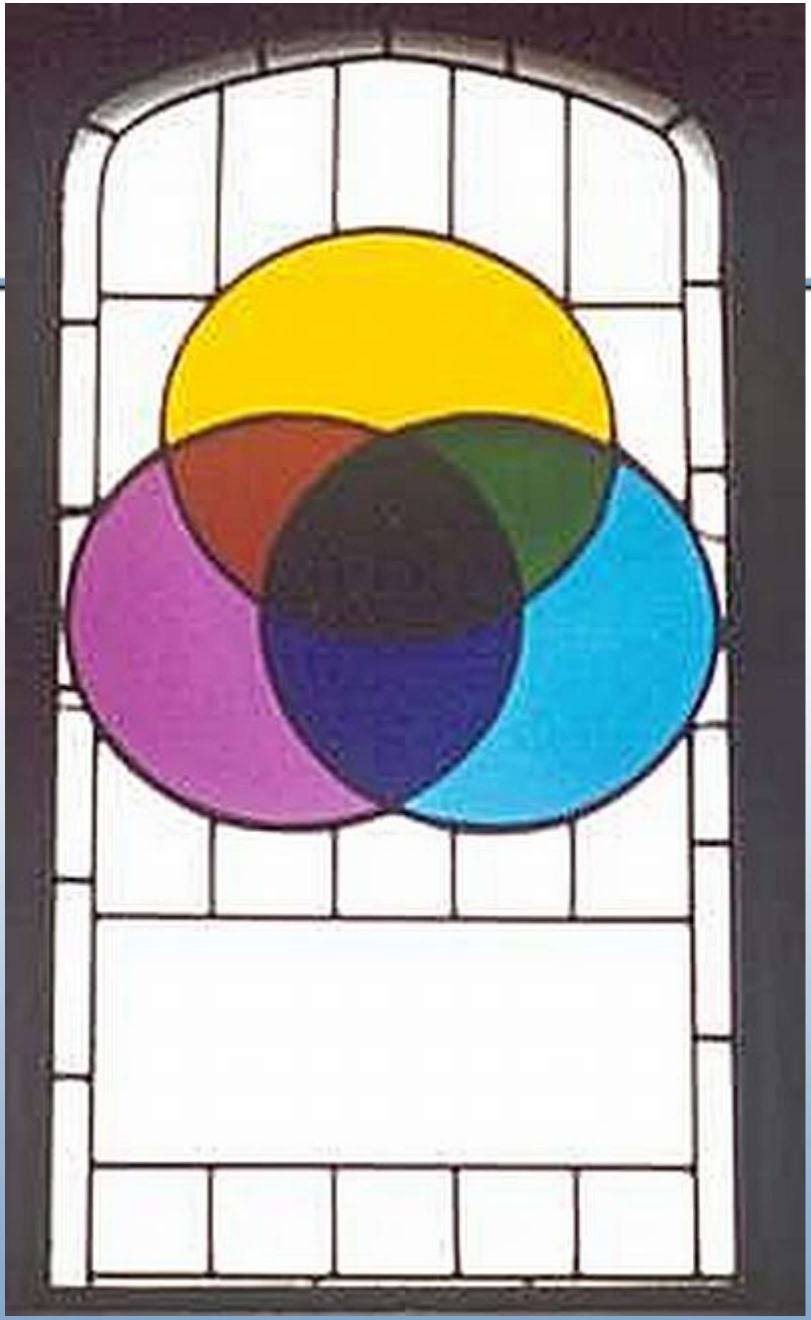
- Lecture notes
- Seminar handouts

are available at

http://gm.softalliance.net/

Advice: download and print lecture notes before the next lecture

Sets



The stained glass in Caius Hall at Cambridge University commemorating John Venn.

Contents

- Notions for sets
- Basic properties of sets
- Venn-Euler diagrams
- Basic set operations
- Membership tables and CSG
- n Sets
- Cartesian product
- Algebra of sets

Set theory

- Creation of one mathematician: Georg Cantor (1845-1918), born in Russia to a Danish father and a Russian mother and spent most of his life in Germany
- Great importance to the modern formulation of many topics of continuous and discrete mathematics

Georg Cantor
1845-1918

Notion of a Set

- Definition by Georg Cantor:
  "A set is a gathering together into a whole of definite, distinct objects of our perception and of our thought – which are called elements of the set."
- More simple "intuitive" or "naive" definition:
  A *set* is a type of structure, representing an *unordered* collection of zero or more *distinct* objects (elements).

Notion of a Set

- Naive definitions turned out to be inadequate for formal mathematics
- Notion of a set is taken as an undefined primitive in axiomatic set theory
- The most basic properties are
  - a set "has" elements
  - two sets are equal if and only if they have the same elements.
- Set theory deals with operations between, relations among, and statements about sets.
- *All* of mathematics can be defined in terms of some form of set theory.
- Sets are extensively used in computer software systems.

Set Membership

- Sets are denoted with capital letters *S*, *T*, *U*, …
- Elements are denoted with low case letters x, y, z …
- If an object *x* is a member of a set ***A*** , then we denote this relationship as: $x \in A$ which reads "x belongs to A", "x is a member of A" or "x is in A".
- If an object x is not a member of a set A , then we denote this relationship as: $x \notin A$ which reads " x does not belong to A ", " x is not a member of A " or " x is not in A ".
- The symbol "∊ " was introduced by the Italian mathematician Giuseppe Peano in 1888, derived from the first letter of the Greek word "ειναι" meaning "is".

Defining a Set

- We may define a particular set in two distinct ways:
  - listing all the members
  - by membership rule or semantic description

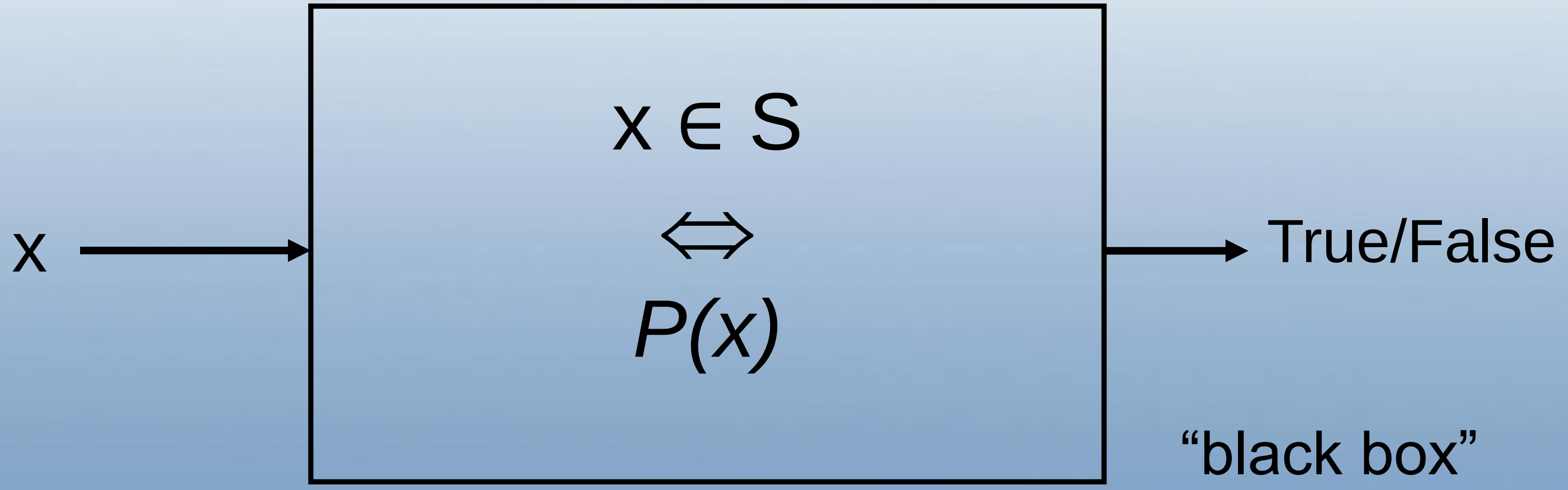
List of set members

- A = {2 ,3, 6 ,8} *tabular form of the set.*
- B = {x | x is an odd integer} or
  B = {x : x is an odd integer}.
  Here the symbols " | " and " : " are read as "where".

Set membership rule

A more general form (a set-builder form):

$S = \{x \mid P(x)\}$ denotes the set *S* of all the entities (objects) *x* for which the predicate *P(x)* holds true.



Defining a Set

Variation of the set-builder form:

$S = \{x \in A \mid P(x)\}$ denotes the set S of all the elements *x* that belong to the set *A* and for which the predicate *P(x)* holds true.

Example:

$S = \{x \in Z \mid P(x)\}$

where *P(x)* ="x is odd“, denotes the set of odd integers.

Contents

The Empty Set

- A set that contains no elements is called a null set or an empty set and is denoted by the symbol "∅".
  - *If A is the set of all people in the world who are older than 200 years, then A is the empty set, i.e. A = ∅ .*
  - *If* $B = \{x \mid x^2 = 4 \wedge x \text{ is an odd integer}\}$, *then* $B = \emptyset$
- The empty set is the unique set that can be defined as $\emptyset = \{\} = \{x|x \neq x\} = \ldots = \{x|\textbf{False}\}$

Finite and Infinite Sets

- A set is finite if it consists of a specific number of different elements (i.e., if the process of counting its elements can terminate.). Otherwise, the set is infinite.

  Examples:
  - If ***D*** is the set of the days of the week, then ***D*** is a *finite set.*
  - If ***O*** = {1, 3, 5, 7, …} , then ***O*** is an *infinite set*.
  - If ***M*** = {x | x is a mountain of this planet}, then ***M*** is a finite set, even though it may be very difficult to count all the mountains.

Cardinality and Finiteness

- If a set ***S*** has *n* elements (where *n* is non-negative integer), then we say that S has cardinality *n*.
- |*S*| (read "the *cardinality* of *S*") is a measure of how many different elements *S* has.
- Examples: |{1,2,3}| = 3, |{a,b}| = 2,
  |{{1,2,3},{4,5}}| = 2
- If |*S*|∈**N**, then we say *S* is *finite*. Otherwise, we say *S* is *infinite*.
- Cardinality of the empty set is 0

Power Set

- The *power set* P(*S*) of a set *S* is the set of all subsets of *S*: $P(S) :\equiv \{x \mid x \subseteq S\}$.

Example: P({a,b}) = {∅, {a}, {b}, {a,b}}.

- Sometimes P(*S*) is written 2^S, because $|P(S)| = 2^{|S|}$.
- It turns out $\forall S: |P(S)| > |S|$, *e.g.* $|P(\mathbf{N})| > |\mathbf{N}|$. *There are different sizes of infinite sets.*

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Venn-Euler Diagrams

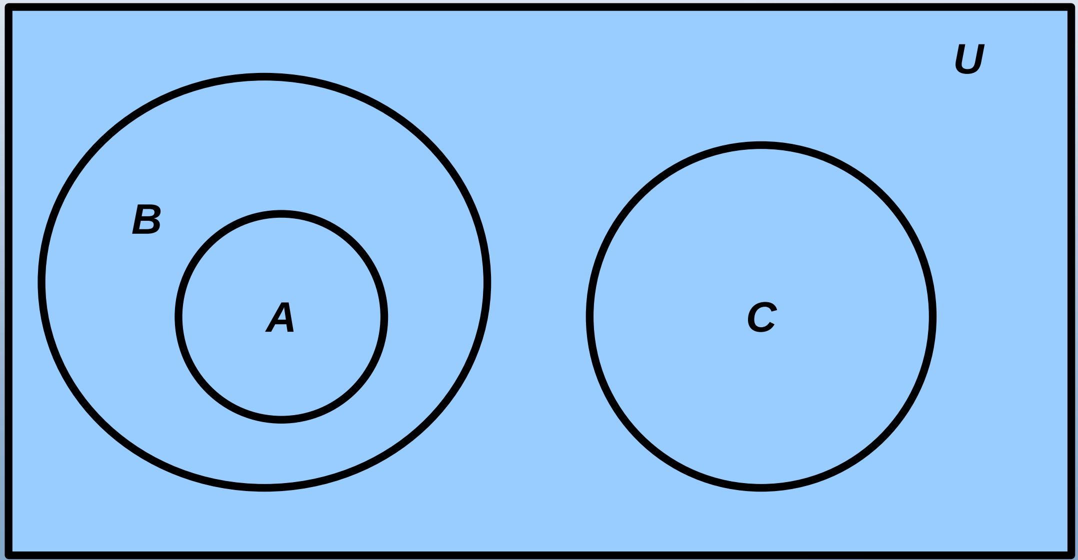
Leonhard Euler
1707-1783

- A Venn-Euler diagram is a pictorial representation of specific sets and their relationships using geometic shapes (sets of points) on the plane to represent them.
- These diagrams were invented by Leonhard Euler and about 100 years later by John Venn. Venn used the term "Eulerian Circles".
- Used to illustrate specific sets and their subsets, and relationships between specific sets.

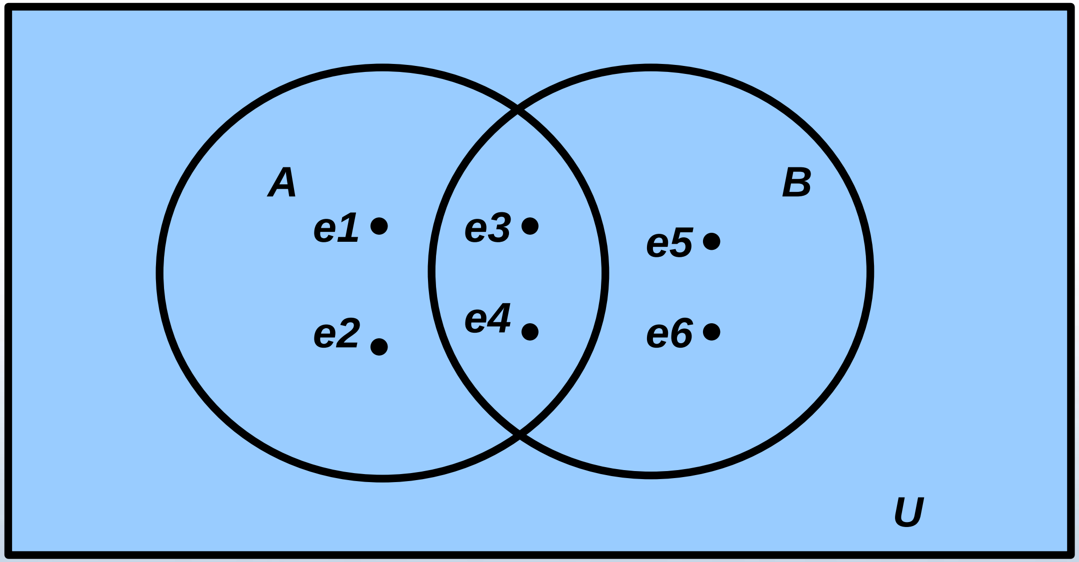
John Venn
1834-1923

Venn-Euler Diagrams

Example:
The universal set U represents all animals,
C represents the set of all camels,
B represents the set of all birds
A represents the set of all albatrosses.
The Venn diagram represents the relationship of these sets.



Venn-Euler Diagrams



Example:

A = {e1 ,e2 ,e3 ,e4}

B = {e3 ,e4 ,e5 ,e6}

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Basic Set Operations: Union

The ***union*** of sets ***A*** and ***B*** is the set of elements that belong to set ***A*** or to set ***B*** or to both sets. We denote the union of sets ***A*** and B by ***A*** ∪ ***B*** , which reads “ ***A*** *union* ***B*** ”.

- $A \cup B = \{x \mid x \in A \lor x \in B\}$

Example: if ***A*** = {a , b , c , d } and B = {c , d , e , f } then ***A*** ∪ ***B*** = {a , b , c , d, e , f }.

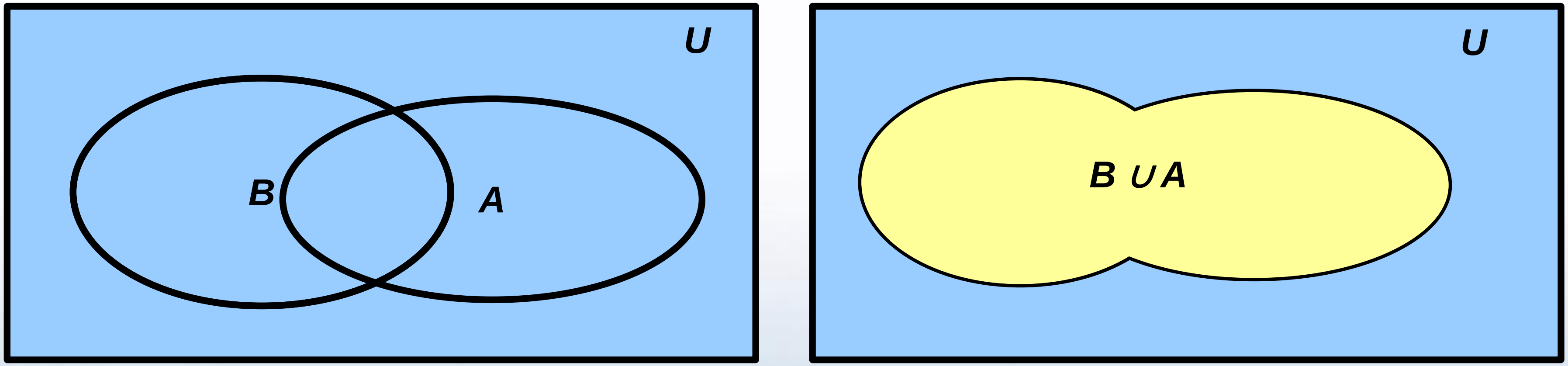
- The union operation is commutative

$$A \cup B = B \cup A$$

- Both sets are subsets of their union

$$A \subseteq (A \cup B) \text{ and } B \subseteq (A \cup B) .$$

Basic Set Operations: Union

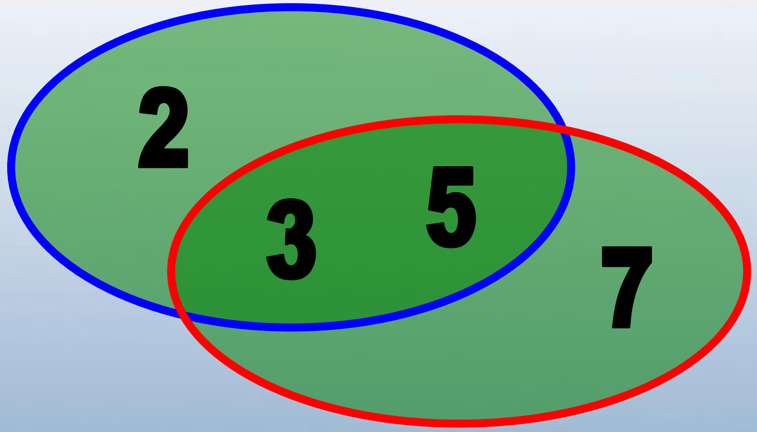


Venn diagram for the ***union*** of sets ***B*** and ***A***

$$B \cup A$$

Union Examples

- $\{a,b,c\} \cup \{2,3\} = \{a,b,c,2,3\}$
- $\{2,3,5\} \cup \{3,5,7\} = \{2,3,5,3,5,7\} = \{2,3,5,7\}$



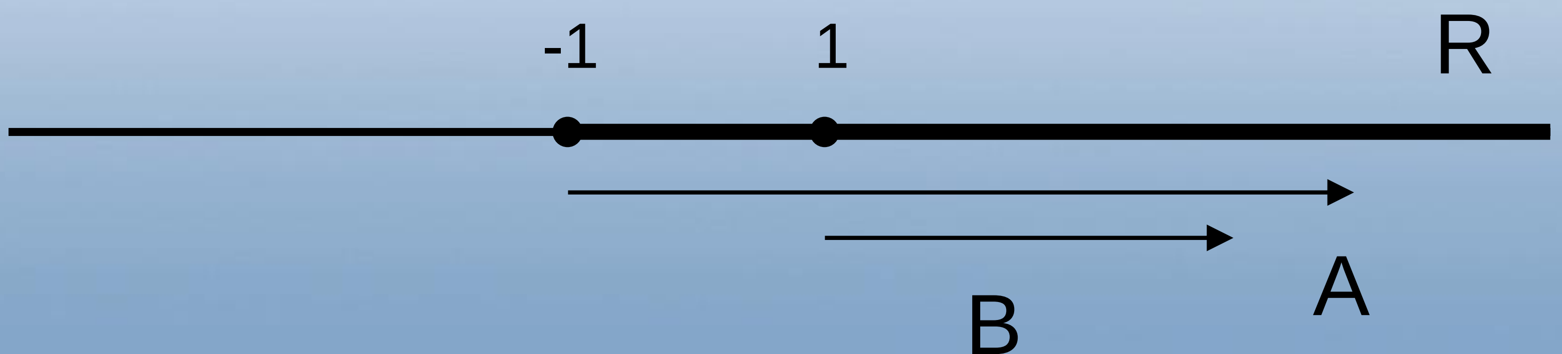
Union Examples

$$A = \{x \in R \mid x \geq -1\}$$

$$B = \{x \in R \mid x \geq 1\}$$

$$A \cup B = \{x \in R \mid x \geq -1 \vee x \geq 1\} =$$

$$\{x \in R \mid x \geq -1\}$$



Intersection operation

The ***intersection*** of sets ***A*** and ***B*** is the set of elements that are common to both sets. We denote the intersection of sets ***A*** and ***B*** by $\boldsymbol{A} \cap \boldsymbol{B}$, which reads " ***A*** *intersection* ***B*** ":

– $\boldsymbol{A} \cap \boldsymbol{B} = \{x \mid x \in \boldsymbol{A} \wedge x \in \boldsymbol{B}\}$

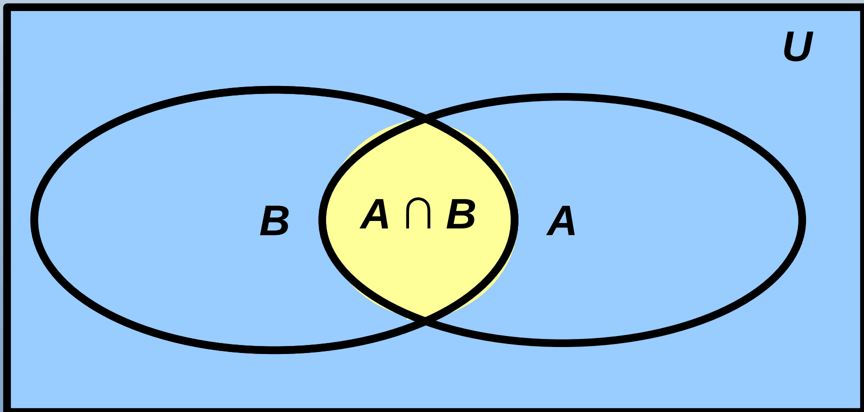
If ***A*** = {a , b , c , d } and B = {c , d , e , f } , then $\boldsymbol{A} \cap \boldsymbol{B}$ = {c , d }.

– The *intersection* is commutative

$$A \cap B = B \cap A.$$

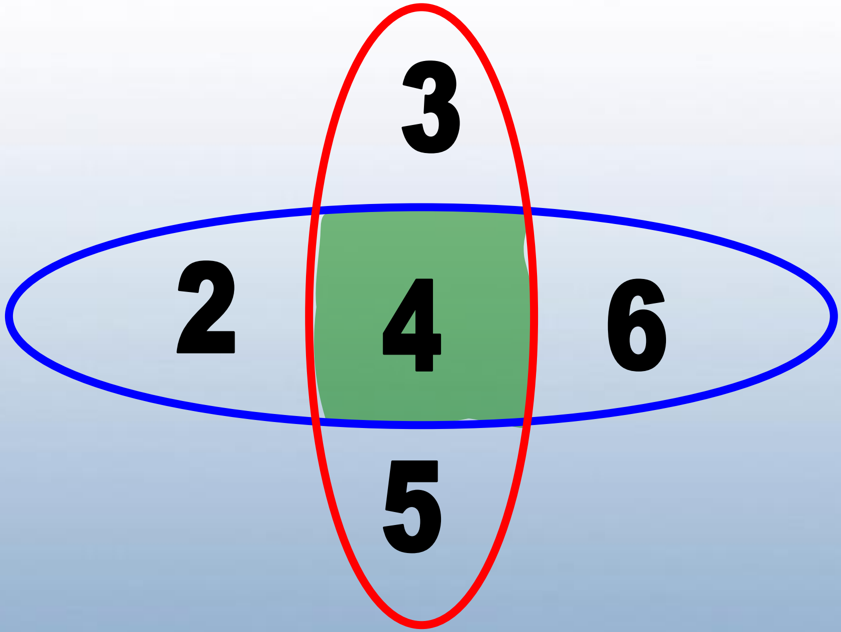
– the *intersection* of two sets is subset of both sets

$(A \cap B) \subseteq A$ and $(A \cap B) \subseteq B$.



Intersection Examples

- $\{a,b,c\} \cap \{2,3\} = \underline{\ \emptyset\ }$
- $\{2,4,6\} \cap \{3,4,5\} = \underline{\ \{4\}\ }$



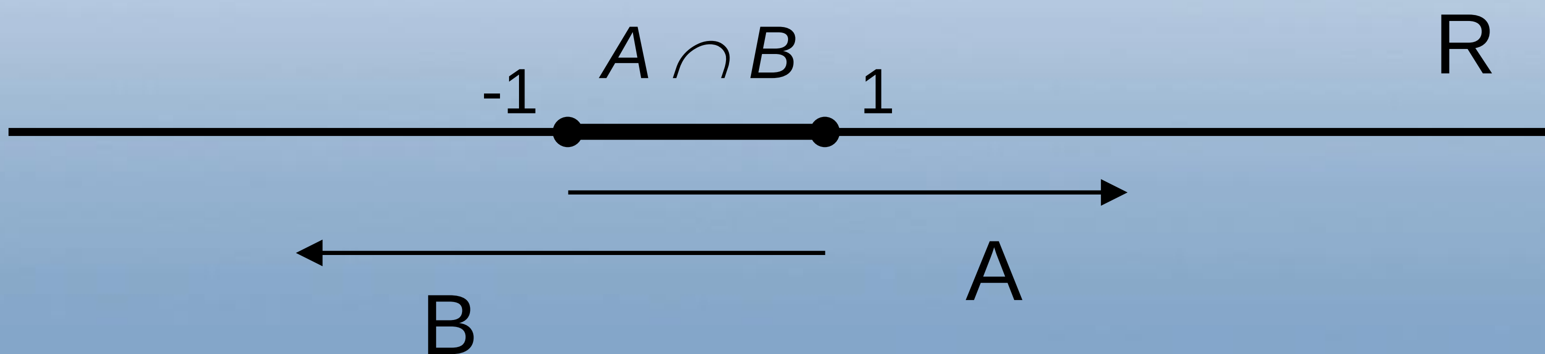
Intersection Examples

$$A = \{x \in R \mid x \geq -1\}$$

$$B = \{x \in R \mid x \leq 1\}$$

$$A \cap B = \{x \in R \mid x \geq -1 \wedge x \leq 1\} =$$

$$\{x \in R \mid -1 \leq x \leq 1\}$$



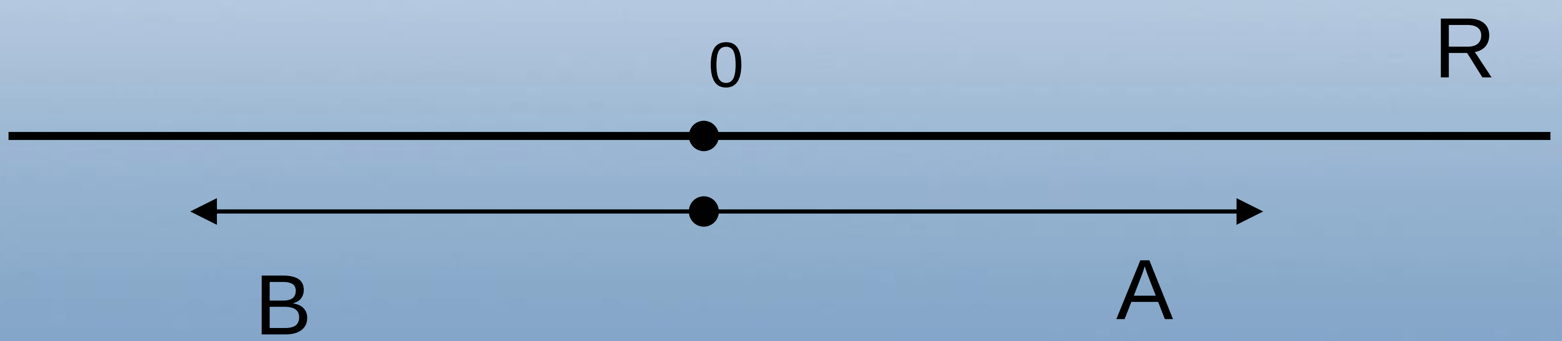
Intersection Examples

$$A = \{x \in R \mid x \geq 0\}$$

$$B = \{x \in R \mid x \leq 0\}$$

$$A \cap B = \{x \in R \mid x \geq 0 \wedge x \leq 0\} =$$

$$\{x \in R \mid x = 0\} = \{0\}$$

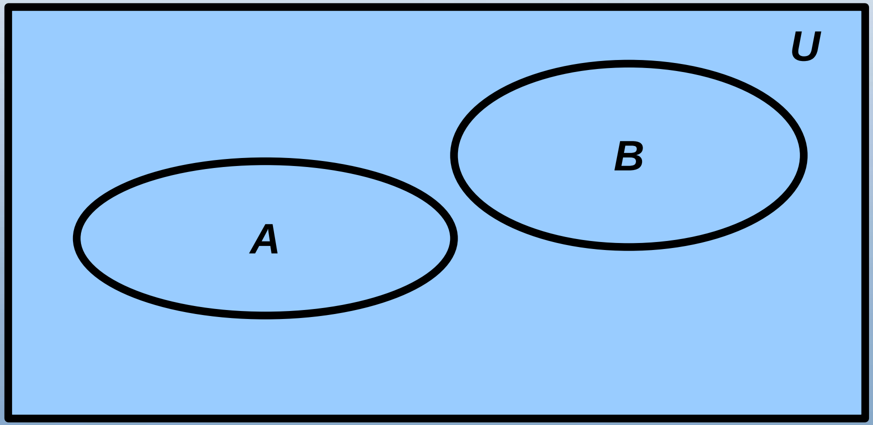


Disjoint Sets Definition

- Two sets *A*, *B* are called *disjoint* (*i.e.*, not joined) if their intersection is empty:

$$A \cap B = \varnothing$$

- Example: the set of even integers is disjoint with the set of odd integers.



The Venn diagram of two disjoint sets.

Inclusion-Exclusion Principle

- How many elements are in $A \cup B$?

  $|A \cup B| = |A| + |B| - |A \cap B|$

  This method of calculation the cardinality is called the inclusion-exclusion principle.
- Example: How many students are on our class list?

  Consider set $E = I \cup M$,

  $I = \{s \mid s$ exists in the attendance sheet$\}$

  $M = \{s \mid s$ exists in the email list$\}$
- Some students may be only in one list

  $|E| = |I \cup M| = |I| + |M| - |I \cap M|$

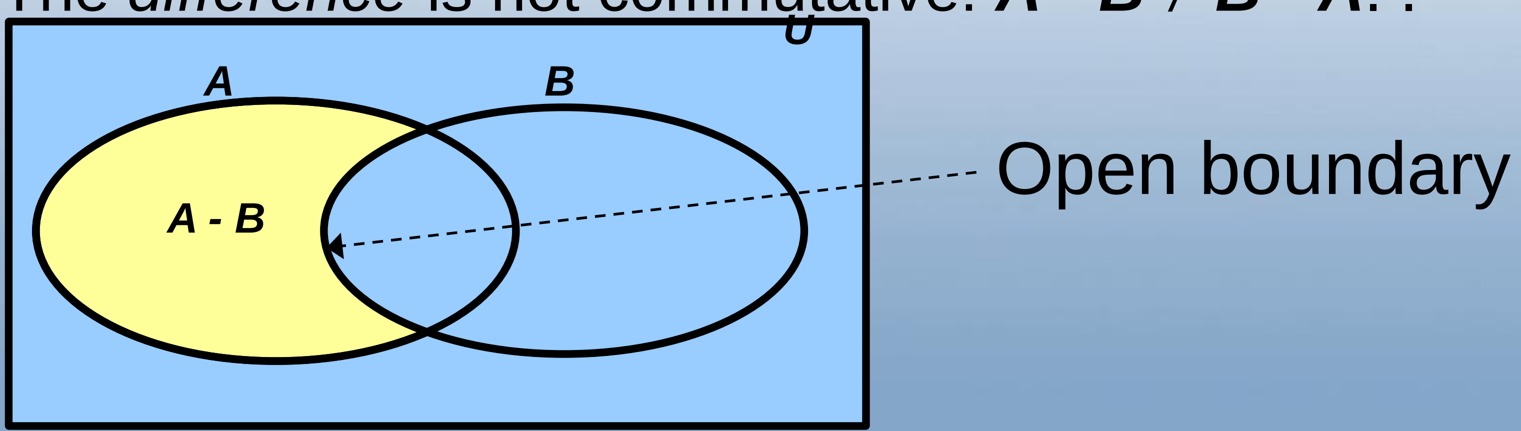
Difference operation

- The ***difference*** of sets ***A*** and ***B*** (subtraction of ***B*** from ***A***) is the set of elements that belong to set A and do not belong to set B. We denote the *difference* of sets ***A*** and ***B*** by ***A - B*** or ***A \ B*** ,

  $A - B = \{x \mid x \in A \wedge x \notin B\}$

  Example: If ***A*** = {a , b , c , d } and B = {c , d , e , f } , then ***A - B*** = {a , b }.

  – The *difference* is not commutative: ***A - B*** ≠ ***B*** - ***A***. .



Difference Examples

- $\{1,2,3,4,5,6\} - \{2,3,5,7,9,11\} =$ $\{1,4,6\}$
- $\mathbf{Z} - \mathbf{N} = \{\ldots, -1, 0, 1, 2, \ldots\} - \{1, 2, \ldots\}$

  $= \{x \mid x \text{ is an integer but not a natural}\}$

  $= \{\ldots, -3, -2, -1, 0\}$

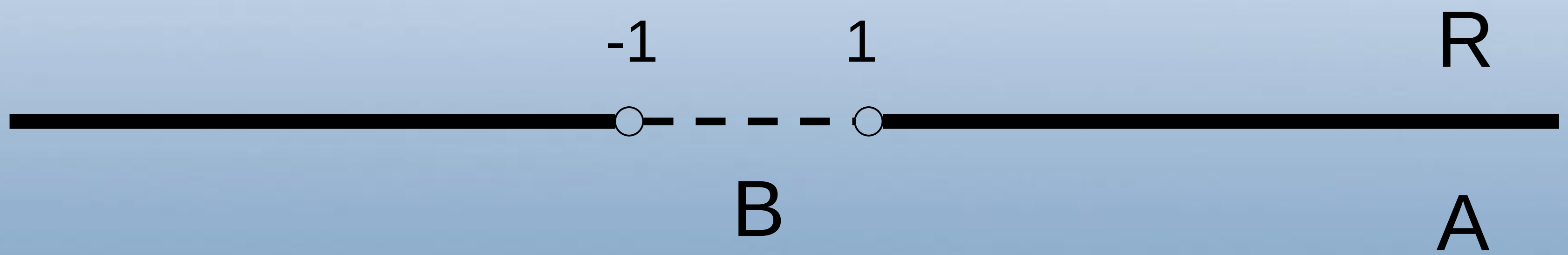
Difference Examples

$$A = \{x \in R\}$$

$$B = \{x \in R \mid -1 \leq x \leq 1\}$$

$$A - B = \{x \in R \mid \neg(-1 \leq x \leq 1)\}$$

$$\{x \in R \mid x < -1 \lor x > 1\}$$



Difference Examples

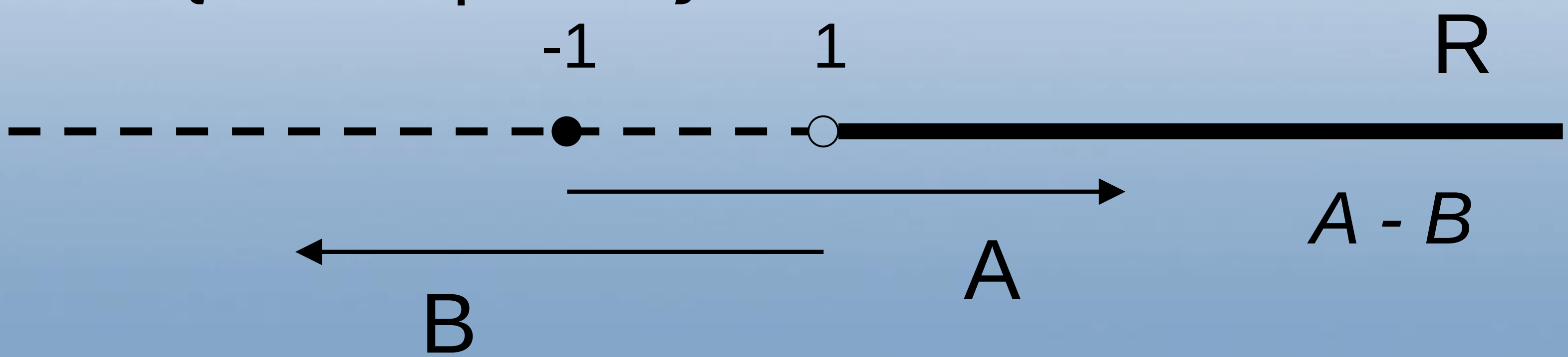
$A = \{x \in R \mid x \geq -1\}$

$B = \{x \in R \mid x \leq 1\}$

$A - B = \{x \in R \mid x \geq -1 \wedge \neg(x \leq 1)\} =$

$\{x \in R \mid x \geq -1 \wedge x > 1\} =$

$\{x \in R \mid x > 1\}$



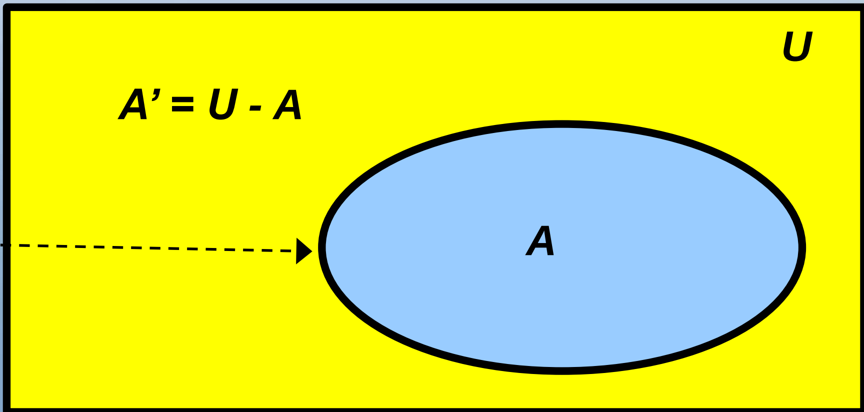
Set Complements

- When the context clearly defines the universal set U, we say that for any set $A \subseteq U$, the *complement* of A, written $\overline{A}$ or A' or $\neg A$ is the complement of A with respect to U: $A' = U - A$

Example: If $U = \mathbf{N}$, $A = \{3,5\}$

$A' = \{1, 2, 4, 6, 7\ldots\}$

Open boundary

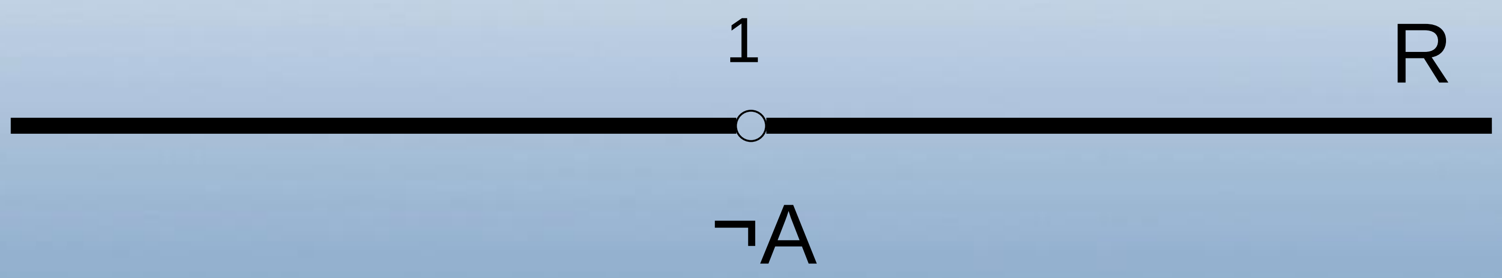


Set Complement Example

$$A = \{x \in R \mid x=1\}$$

$$\neg A = \{x \in R \mid \neg (x=1)\} =$$

$$\{x \in R \mid x < 1 \vee x > 1\}$$



Symmetric Difference

The *symmetric* ***difference*** of sets ***A*** and ***B*** is the set of elements that belong to one of the sets A or B and do not belong to both sets:

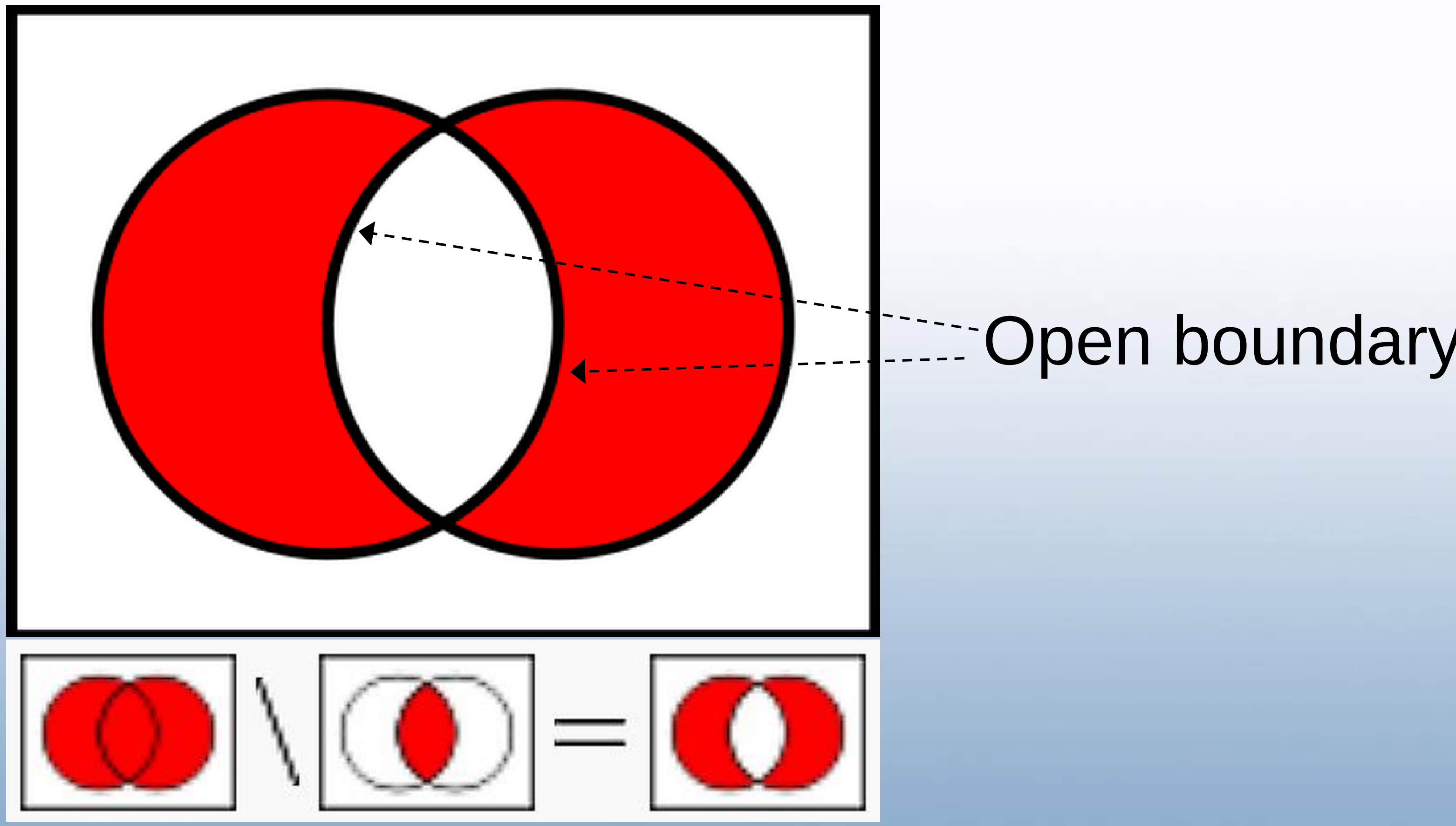
$$\boldsymbol{A} \vartriangle \boldsymbol{B} = \{x \mid (x \in \boldsymbol{A} \wedge x \notin \boldsymbol{B}) \vee (x \in \boldsymbol{B} \wedge x \notin \boldsymbol{A}) \} =$$

$$\{x \mid x \in \boldsymbol{A} \oplus x \in \boldsymbol{B}\}$$

Example:

If $\boldsymbol{A} = \{a , b , c , d \}$ and $B = \{c , d , e , f \}$, then $\boldsymbol{A} \vartriangle \boldsymbol{B} = \{a , b, e, f\}$.

Symmetric Difference



$$A \bigtriangleup B = (A \cup B) - (A \cap B)$$

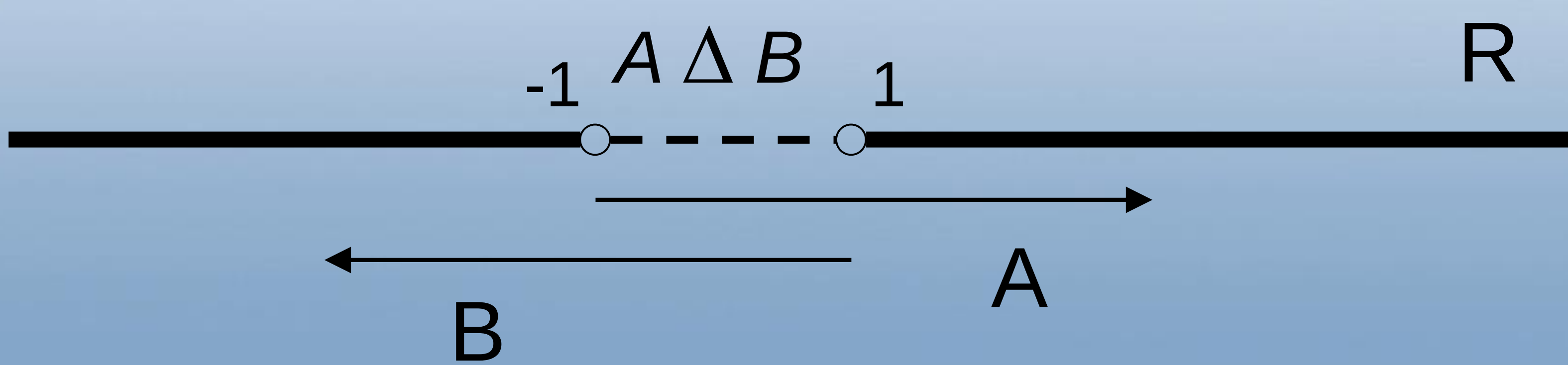
Symmetric Difference Example

$$A = \{x \in R \mid x \geq -1\}$$

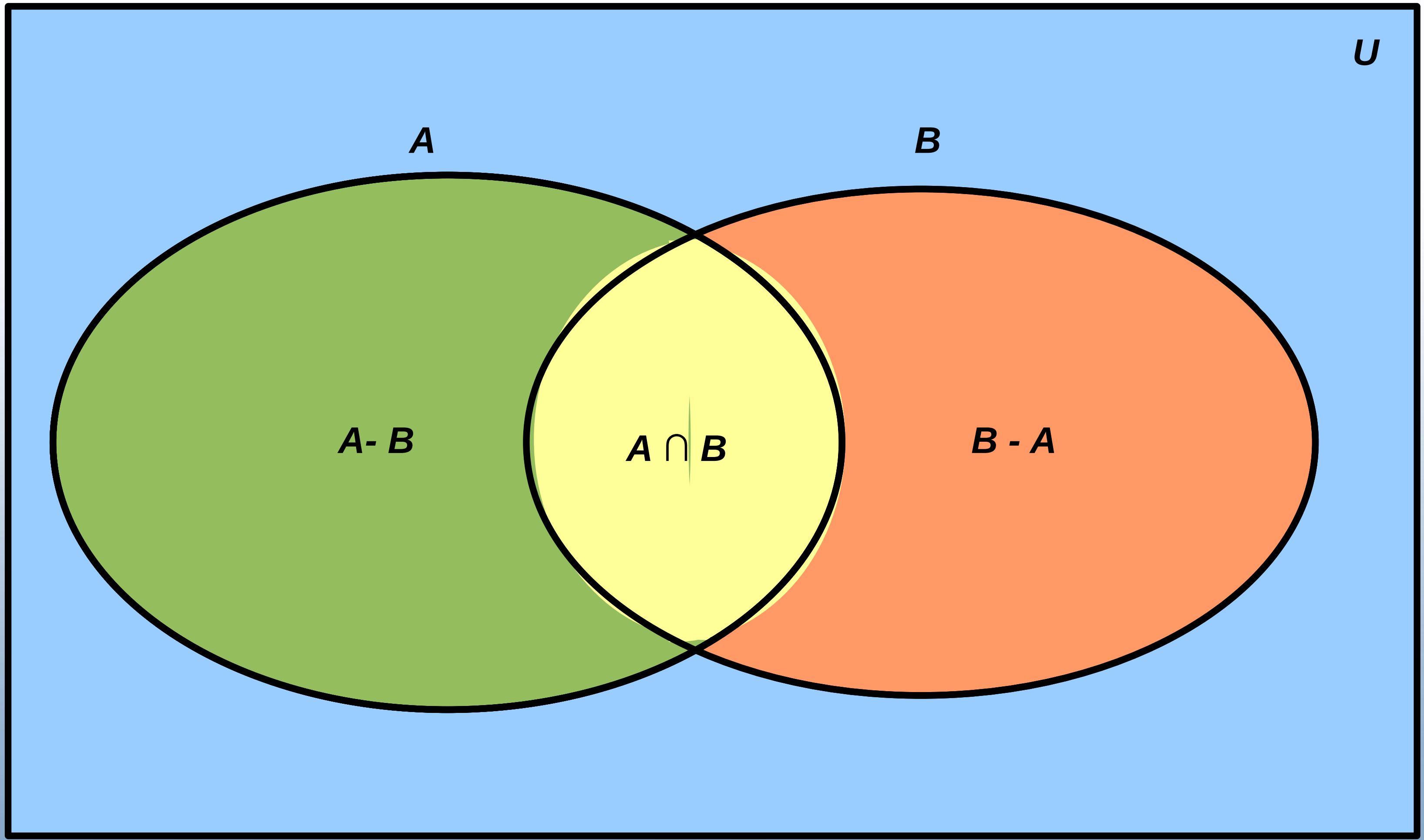
$$B = \{x \in R \mid x \leq 1\}$$

$$A \,\Delta\, B = \{x \in R \mid (x \geq -1 \vee x \leq 1) - (-1 \leq x \leq 1)\} =$$

$$\{x \in R \mid x < -1 \vee x > 1\}$$



Basic Set Operations: summary



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Set Membership Tables

- Just like truth tables for propositional logic.
- Columns for different set expressions.
- Rows for all combinations of memberships in constituent sets.
- 2^n rows for n constituent sets
- Use “1” to indicate membership in the derived set, “0” for non-membership.
- Prove equivalence of set expressions with identical columns.

Membership Table Example

Prove $(A \cup B) - B = A - B$.

A	B	$A \cup B$	$(A \cup B) - B$	$A - B$
0	0	0	0	0
0	1	1	0	0
1	0	1	1	1
1	1	1	0	0

Membership Table Exercise

Prove $(A \cup B) - C = (A - C) \cup (B - C)$.

A B C	$A \cup B$	$(A \cup B) - C$	$A - C$	$B - C$	$(A - C) \cup (B - C)$
0 0 0					
0 0 1					
0 1 0	1				
0 1 1	1				
1 0 0	1				
1 0 1	1				
1 1 0	1				
1 1 1	1				

Membership Table Exercise

Prove $(A \cup B) - C = (A - C) \cup (B - C)$.

$A\ B\ C$	$A \cup B$	$(A \cup B) - C$	$A - C$	$B - C$	$(A - C) \cup (B - C)$
0 0 0					
0 0 1					
0 1 0	1	1			
0 1 1	1				
1 0 0	1	1			
1 0 1	1				
1 1 0	1	1			
1 1 1	1				

Membership Table Exercise

Prove $(A \cup B) - C = (A - C) \cup (B - C)$.

A B C	$A \cup B$	$(A \cup B) - C$	$A - C$	$B - C$	$(A - C) \cup (B - C)$
0 0 0					
0 0 1					
0 1 0	1	1		1	
0 1 1	1				
1 0 0	1	1	1		
1 0 1	1				
1 1 0	1	1	1	1	
1 1 1	1				

Membership Table Exercise

Prove $(A \cup B) - C = (A - C) \cup (B - C)$.

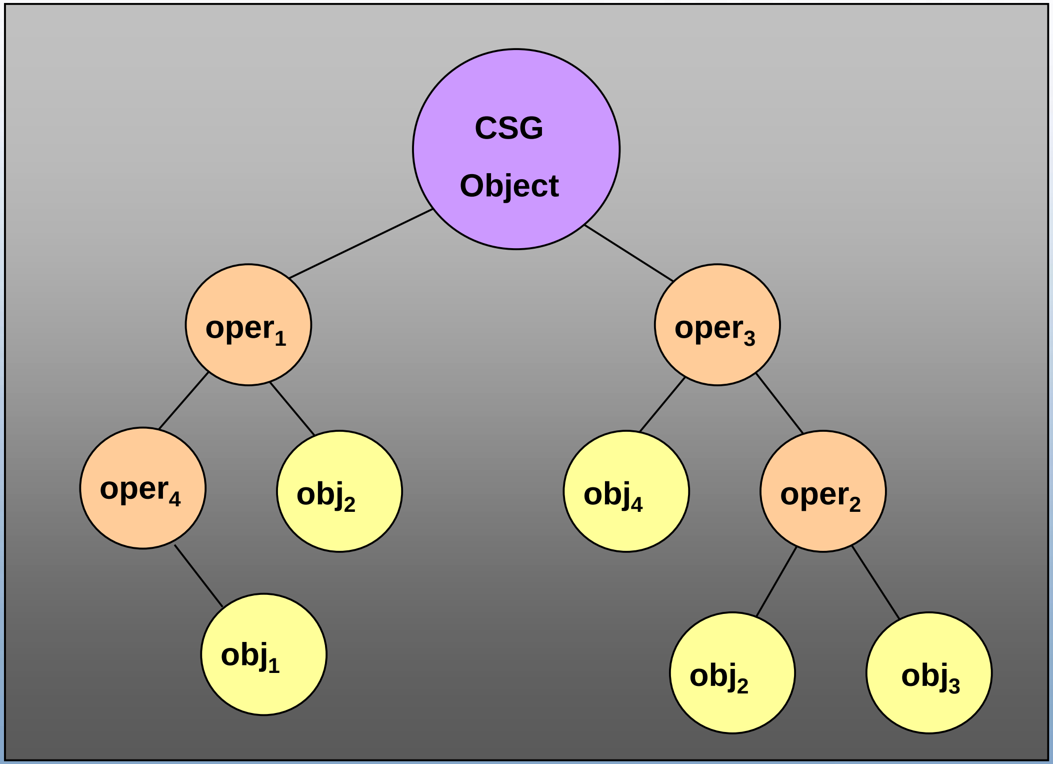
A B C	$A \cup B$	$(A \cup B) - C$	$A - C$	$B - C$	$(A - C) \cup (B - C)$
0 0 0					
0 0 1					
0 1 0	1	1		1	1
0 1 1	1				
1 0 0	1	1	1		1
1 0 1	1				
1 1 0	1	1	1	1	1
1 1 1	1				

Constructive Solid Geometry (CSG)

- CSG is based on a set of 3D solid primitives and set-theoretic operations
- Traditional primitives: block, cylinder, cone, sphere, torus
- Operations; union, intersection, difference + translation and rotation

CSG tree

- A complex solid is represented with a binary tree usually called CSG tree



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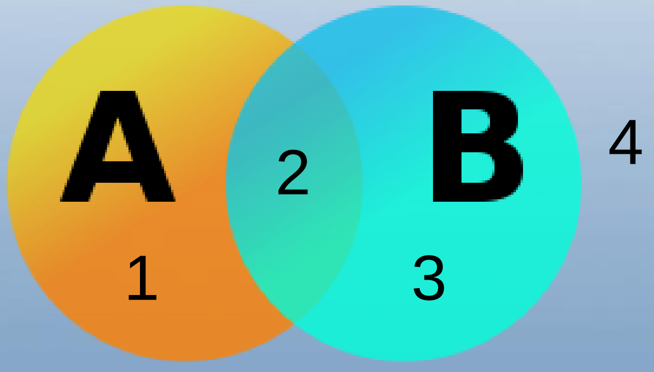
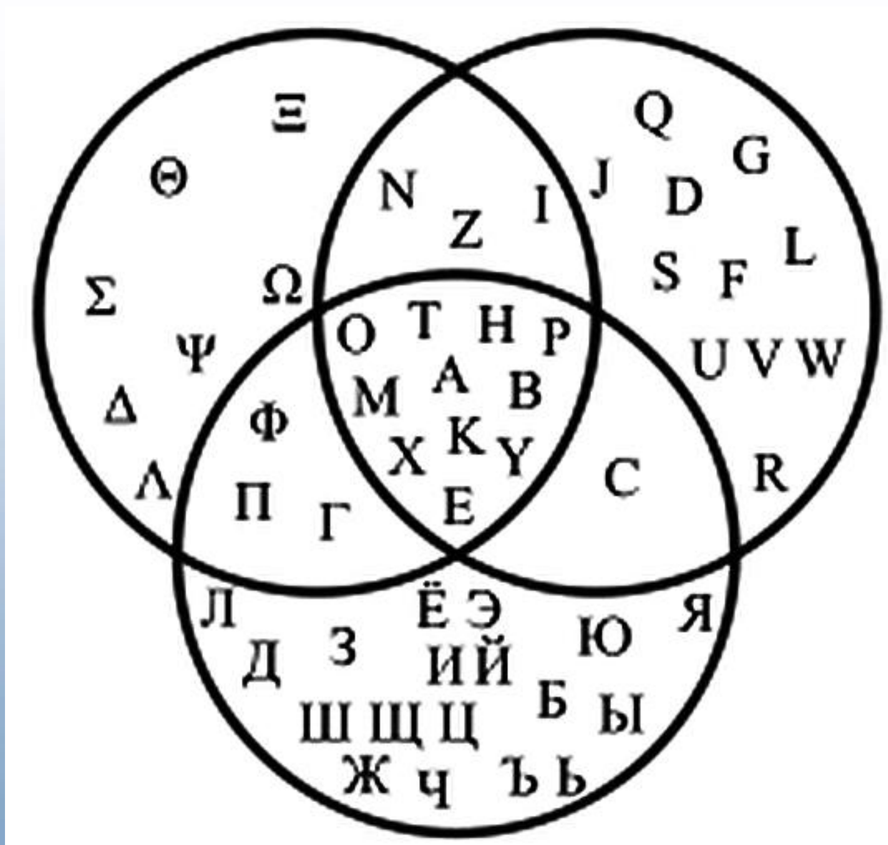
Venn Diagrams for n Sets

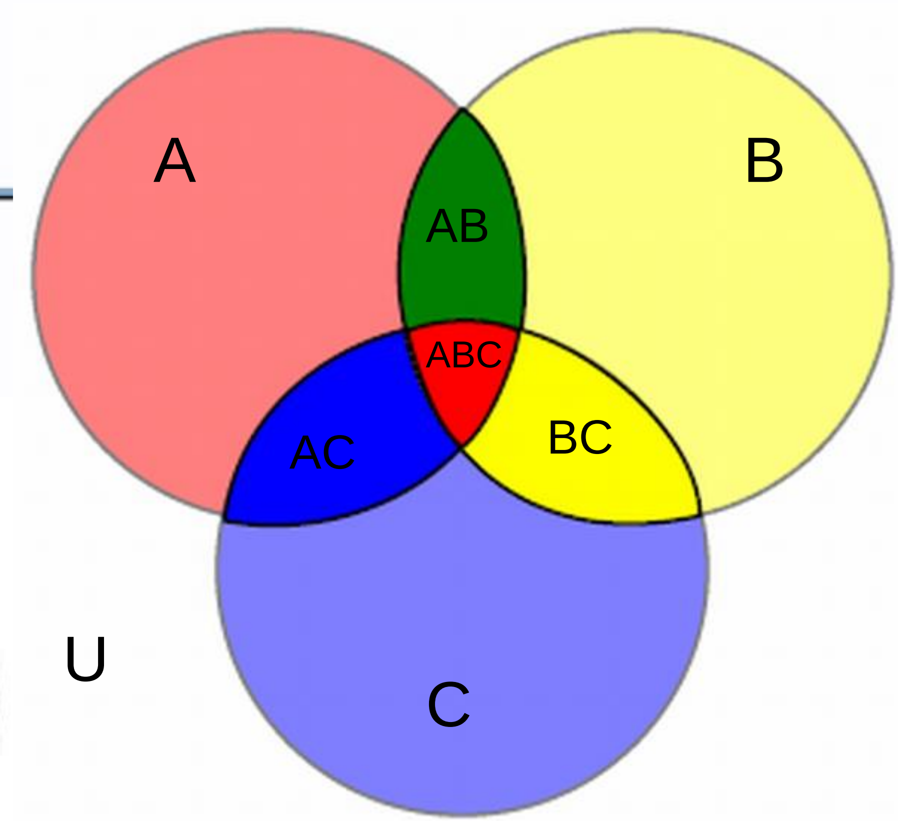
- Venn diagram for *n* sets must contain all 2^n hypothetically possible zones that correspond to all combinations of inclusion or exclusion in each of the component sets.
- 2^n zones correspond to the number of rows in the set membership table:
- n=2, 4 zones;
- n=3, 8 zones;
- n=4, 16 zones, etc.



Venn Diagrams for n Sets

n=3, 8 zones

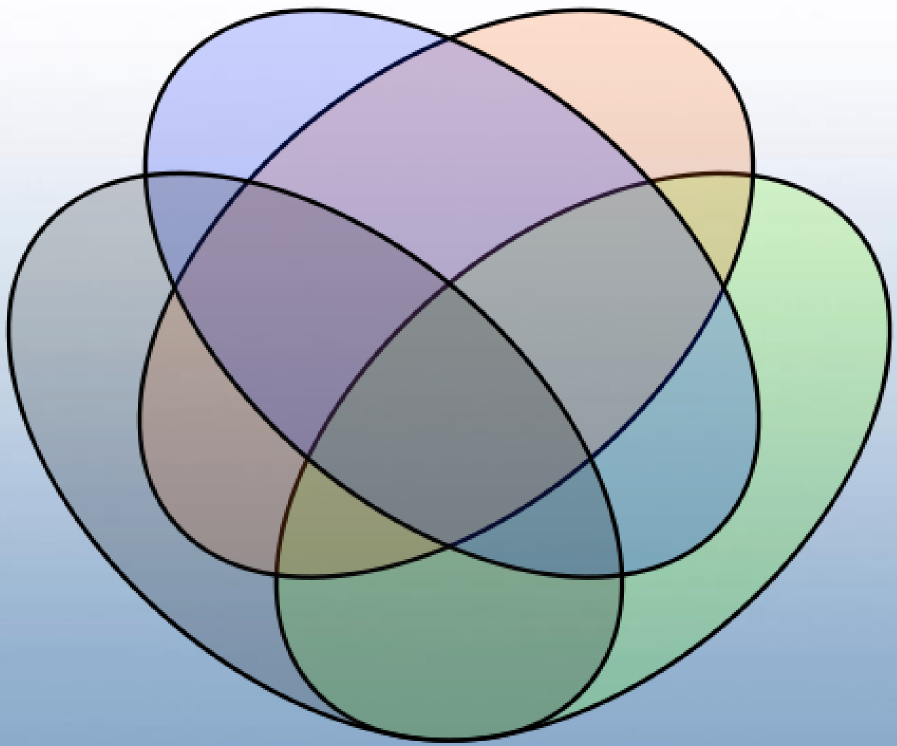




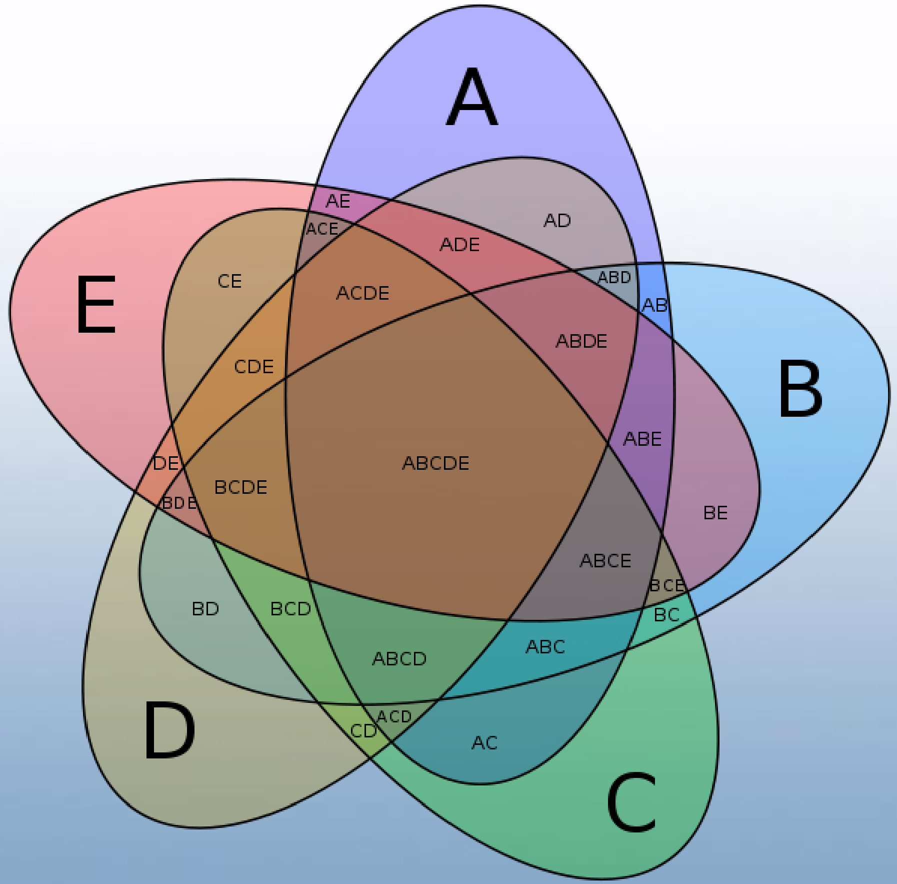
Venn diagram: intersections of the Greek, Latin and Russian alphabets

Venn Diagrams for n Sets

n=4, 16 zones

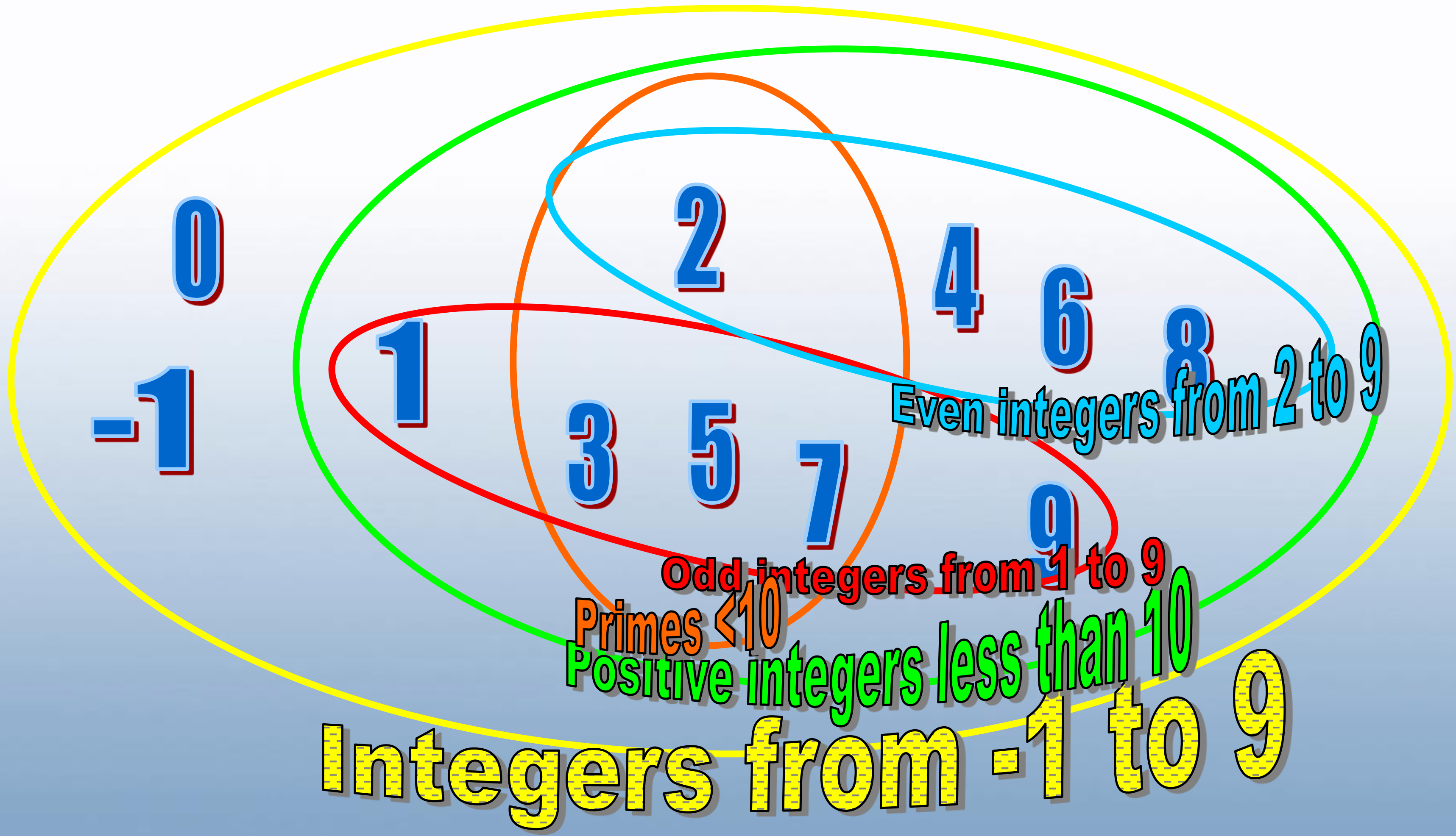


n=5, 32 zones



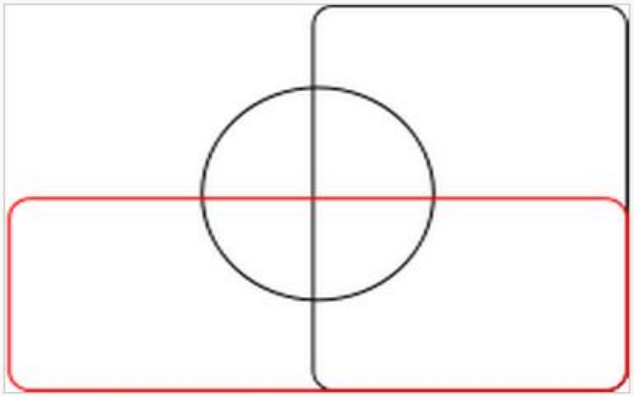
Devised by Branko Grünbaum

Venn Diagrams for n Sets

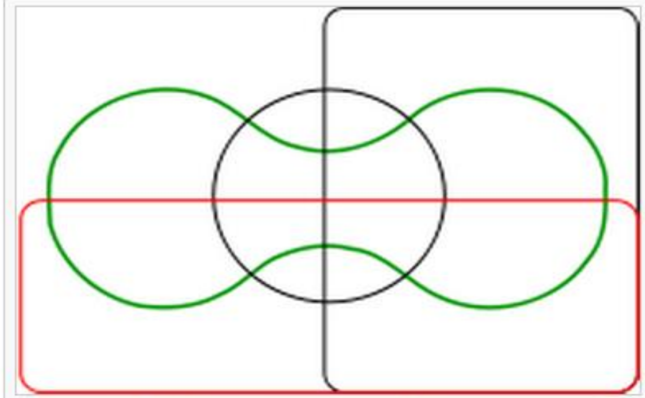


Venn Diagrams for n Sets

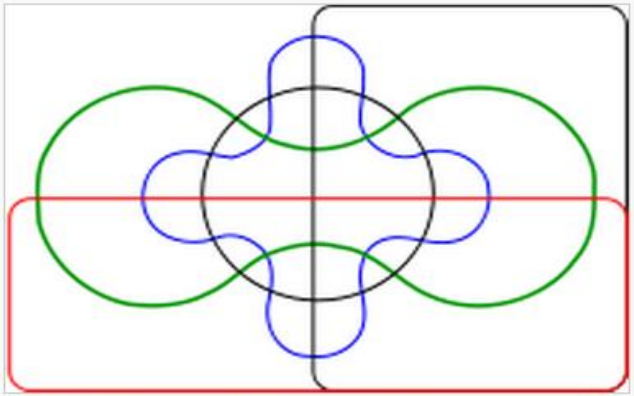
Venn diagrams devised by Anthony Edwards for n = 3, 4, 5, 6



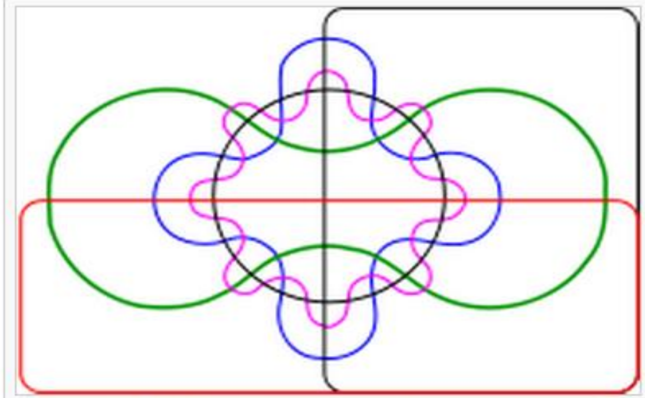
Edwards' Venn diagram of three sets



Edwards' Venn diagram of four sets



Edwards' Venn diagram of five sets



Edwards' Venn diagram of six sets

Generalized Unions & Intersections

- Since union & intersection are commutative and associative, we can extend them from operating on *ordered pairs* of sets (A,B) to operating on sequences of sets ($A_1,\ldots,A_n$), or even on unordered *sets* of sets,
  $\Psi = \{A \mid P(A)\}$ (for some property P).

(This is just like using Σ when adding up large or variable numbers of numbers)

Generalized Union

- Binary union operator: $A \cup B$
- *n*-ary union:
  $A \cup A_2 \cup \ldots \cup A_n :\equiv ((\ldots((A_1 \cup A_2) \cup \ldots) \cup A_n)$
  (grouping & order is irrelevant)
- "Big U" notation:

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n$$

- Or for infinite sets of sets Ψ: $\bigcup_{A \in \Psi} A$

Generalized Intersection

- Binary intersection operator: $A \cap B$
- n-ary union:
  $A_1 \cap A_2 \cap \ldots \cap A_n \equiv ((\ldots((A_1 \cap A_2) \cap \ldots) \cap A_n)$
  (grouping & order is irrelevant)
- “Big Arch” notation:

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$$

- Or for infinite sets of sets Ψ: $\bigcap_{A \in \Psi} A$

Contents

- Notions for sets
- Basic properties of sets
- Venn-Euler diagrams
- Basic set operations
- Membership tables and CSG
- n Sets
- Cartesian product
- Algebra of sets

Tuples

- Sometimes we need to consider ordered collections of objects
- Definition: The ordered n-tuple $(a_1,a_2,\ldots,a_n)$ is the ordered collection with the element a_i being the i-th element for $i=1,2,\ldots,n$
- Two ordered n-tuples $(a_1,a_2,\ldots,a_n)$ and $(b_1,b_2,\ldots,b_n)$ are equal if and only if for every $i=1,2,\ldots,n$ we have $a_i=b_i$ $(a_1,a_2,\ldots,a_n)$
- A 2-tuple $(n=2)$ is called an ordered pair

Cartesian Products of Sets

- For sets *A*, *B*, their *Cartesian product* $A \times B :\equiv \{(a, b) \mid a \in A \wedge b \in B\}$.

  is the set of all possible ordered pairs whose first component is a member of *A* and whose second component is a member of *B*

Example:

$\{a,b\} \times \{1,2\} = \{(a,1),(a,2),(b,1),(b,2)\}$

- Other terms: product set, set direct product, or cross product

René Descartes (1596-1650)

Cartesian Products of Sets

Example:

{John,Mary,Ellen} $\times$ {News,Soap} =

{(John,News), (Mary,News), (Ellen,News), (John,Soap), (Mary,Soap), (Ellen,Soap)}

- Subset of a Cartesian product, R $\subseteq$ A$\times$B is called a relation over the sets A and B.

Example: {(John,News), (Mary,Soap), (Ellen,Soap)} is a relation over sets {John,Mary,Ellen} and {News,Soap}

Cartesian Products of Sets

- Note that
  - for finite *A*, *B*, $|A \times B| = |A|.|B|$
  - the Cartesian product is *not* commutative:
    $\neg \forall A,B: A \times B = B \times A$

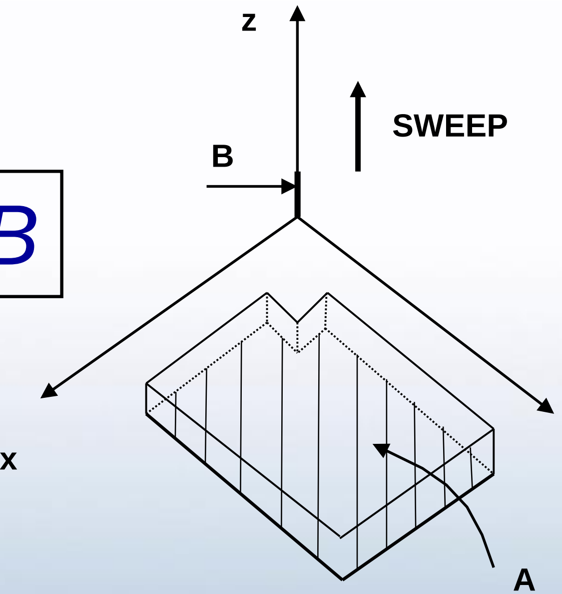
  $A \times B = B \times A$, if $A=\varnothing$ or $B=\varnothing$ or $A=B$
- Cartesian product can be generalized for any n-tuple: Cartesian product of n sets, $A_1, A_2, \ldots, A_n$ is

  $A_1 \times A_2 \times \ldots \times A_n = \{ (a_1, a_2, \ldots, a_n) \mid a_i \in A_i \text{ for } i=1,2,\ldots,n \}$
- Cartesian power of a set $A^n = A \times A \times \ldots \times A$

Sweep as Cartesian Product

- Set of all points visited by an object A moving along a trajectory B is a new solid, called a sweep.
- Translational sweeping (extrusion): 2D area moves along a line normal to the plane of the area.

$A \times B$



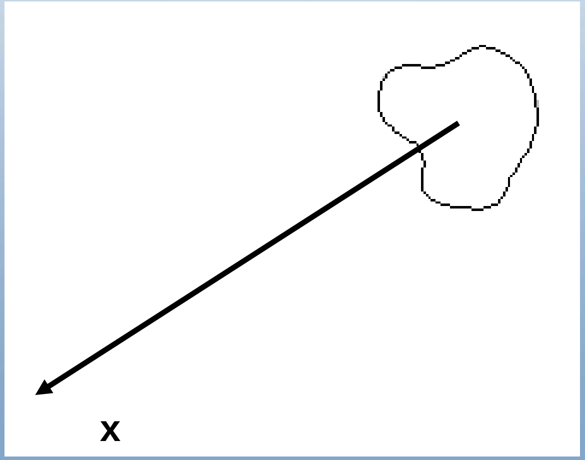


Image by Martin Culpepper, 1999

Review: Set Notations

- Set enumeration {a, b, c}
  and set-builder $\{x|P(x)\}$
- $\in$ relation, and the empty set $\emptyset$.
- Set relations =, $\subseteq$, $\supseteq$, $\subset$, $\supset$, $\not\subset$, etc.
- Cardinality $|S|$
- Power sets P(S)
- Venn diagrams
- Set operations $\cup$, $\cap$, -, $\times$
- Constructive Solid Geometry, sweeping

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Algebra of Sets

U Universal set and its subsets ***A* , *B* , *C***

The Identity Rules:

$$A \cup \emptyset = A$$

$$A \cap U = A$$

$$A \cup U = U$$

$$A \cap \emptyset = \emptyset$$

The Idempotent Rules:

$$(A')' = A$$

$$A \cup A = A$$

$$A \cap A = A$$

The Complement Rules:

$$A \cup A' = U$$

$$A \cap A' = \emptyset$$

$$U' = \emptyset$$

$$\emptyset' = U$$

Algebra of Sets

The Associative Rules:

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

The Distributive Rules:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

The De Morgan Rules:

i) $(A \cup B)' = A' \cap B'$

ii) $(A \cap B)' = A' \cup B'$

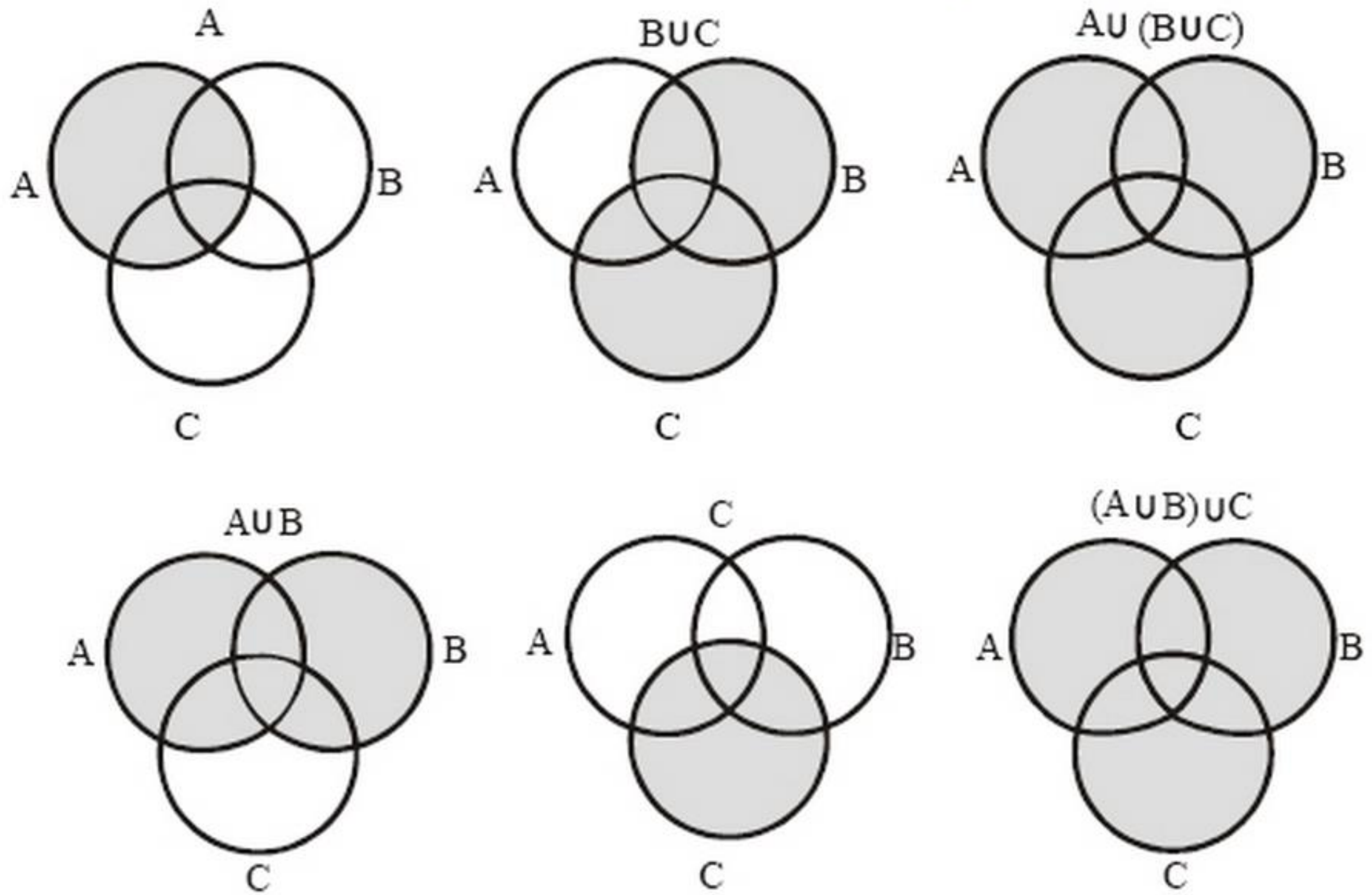
iii) $A – (B \cup C) = (A - B) \cap (A-C)$

iv) $A – (B \cap C) = (A - B) \cup (A-C)$

Associative Rules

$A \cup (B \cup C) = (A \cup B) \cup C$

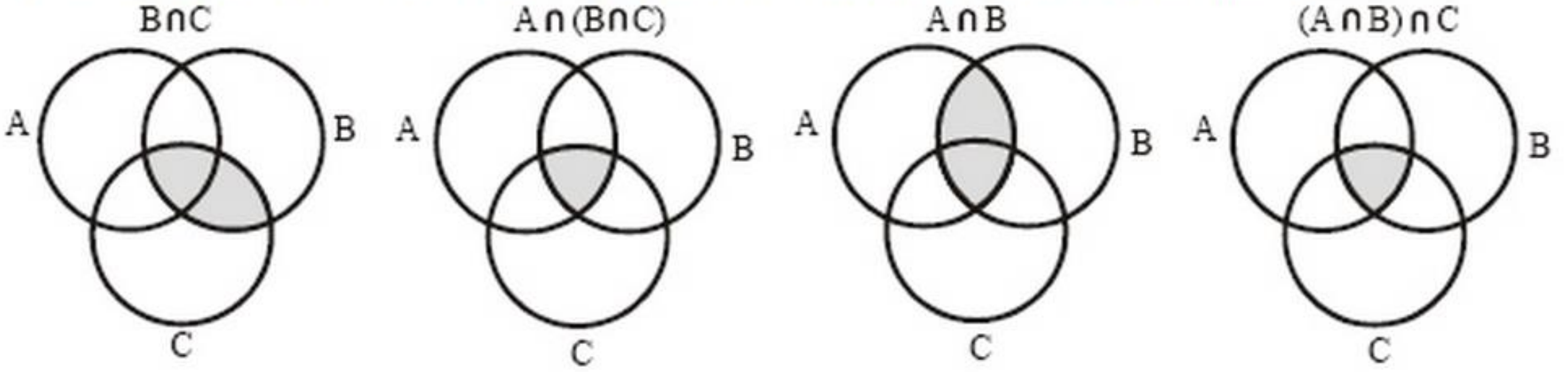
Verification of the associative law for the union of sets using Venn diagrams:



Algebra of Sets

$A \cap (B \cap C) = (A \cap B) \cap C$

Verification of the associative law for intersection of sets using Venn diagrams"



Algebra of Sets

The Associative Rules:

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

The Distributive Rules:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

The De Morgan Rules:

i) $(A \cup B)' = A' \cap B'$

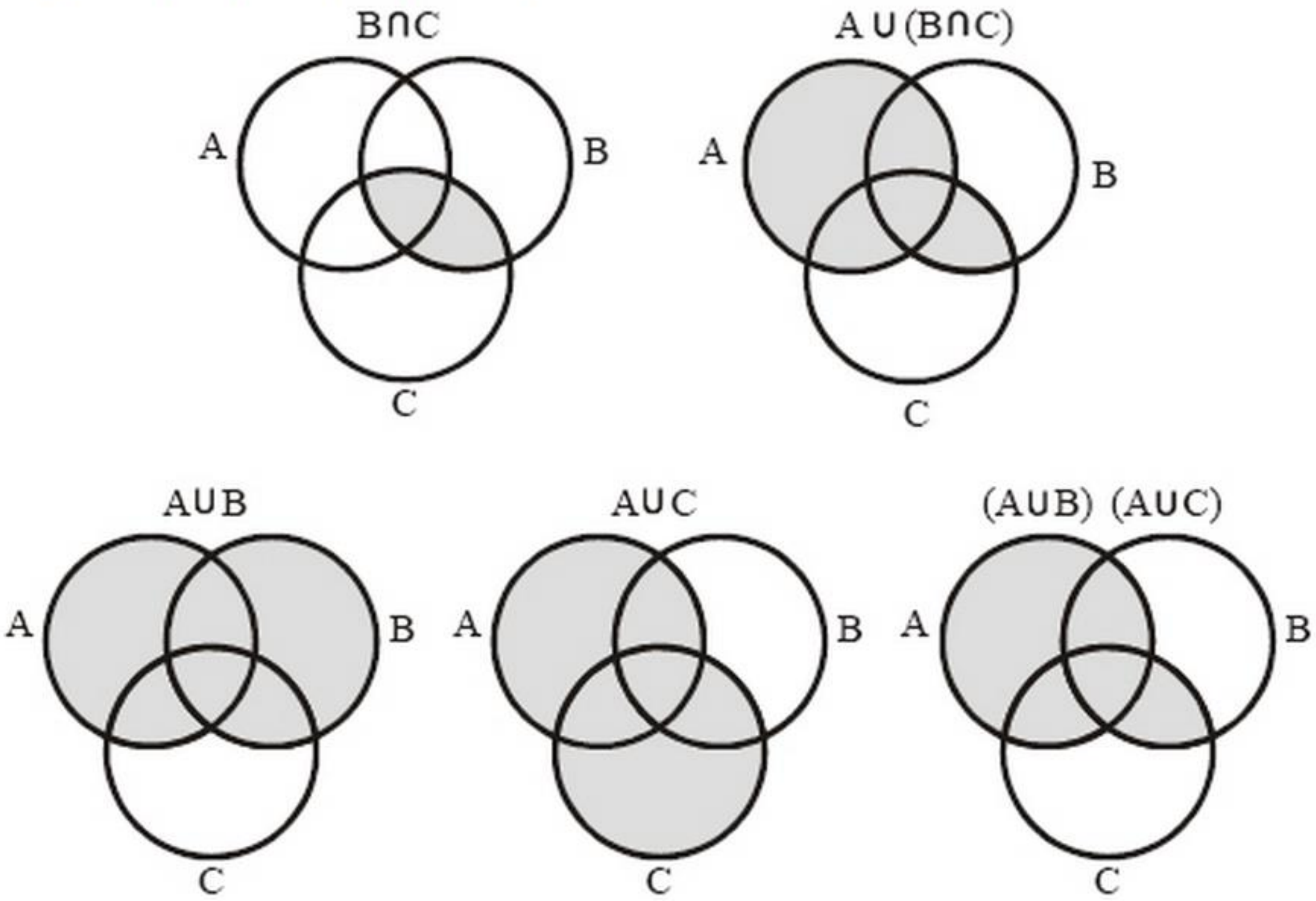
ii) $(A \cap B)' = A' \cup B'$

iii) $A - (B \cup C) = (A - B) \cap (A - C)$

iv) $A - (B \cap C) = (A - B) \cup (A - C)$

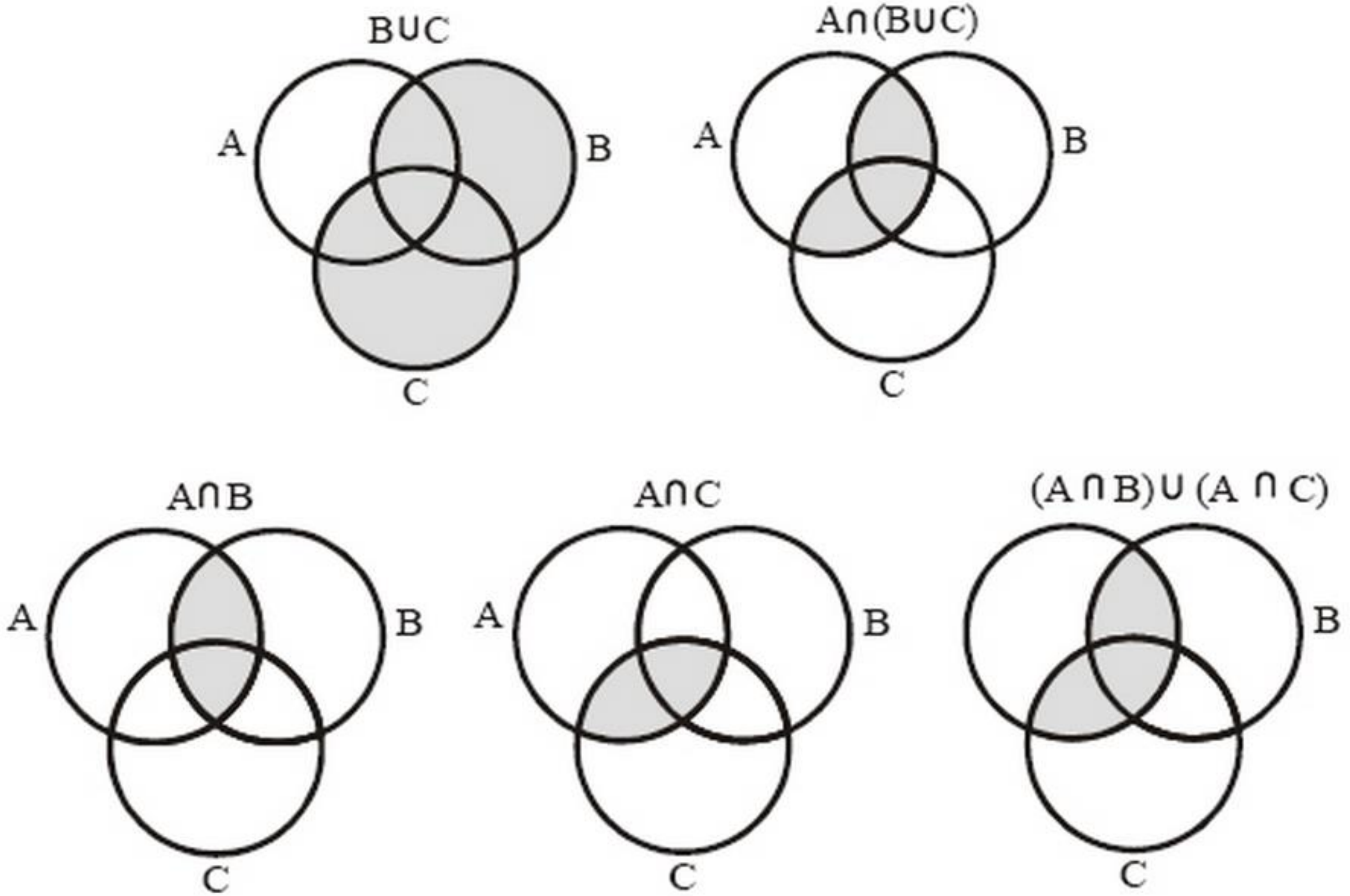
Distributive Rules

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



Algebra of Sets

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



Algebra of Sets

The Associative Rules:

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

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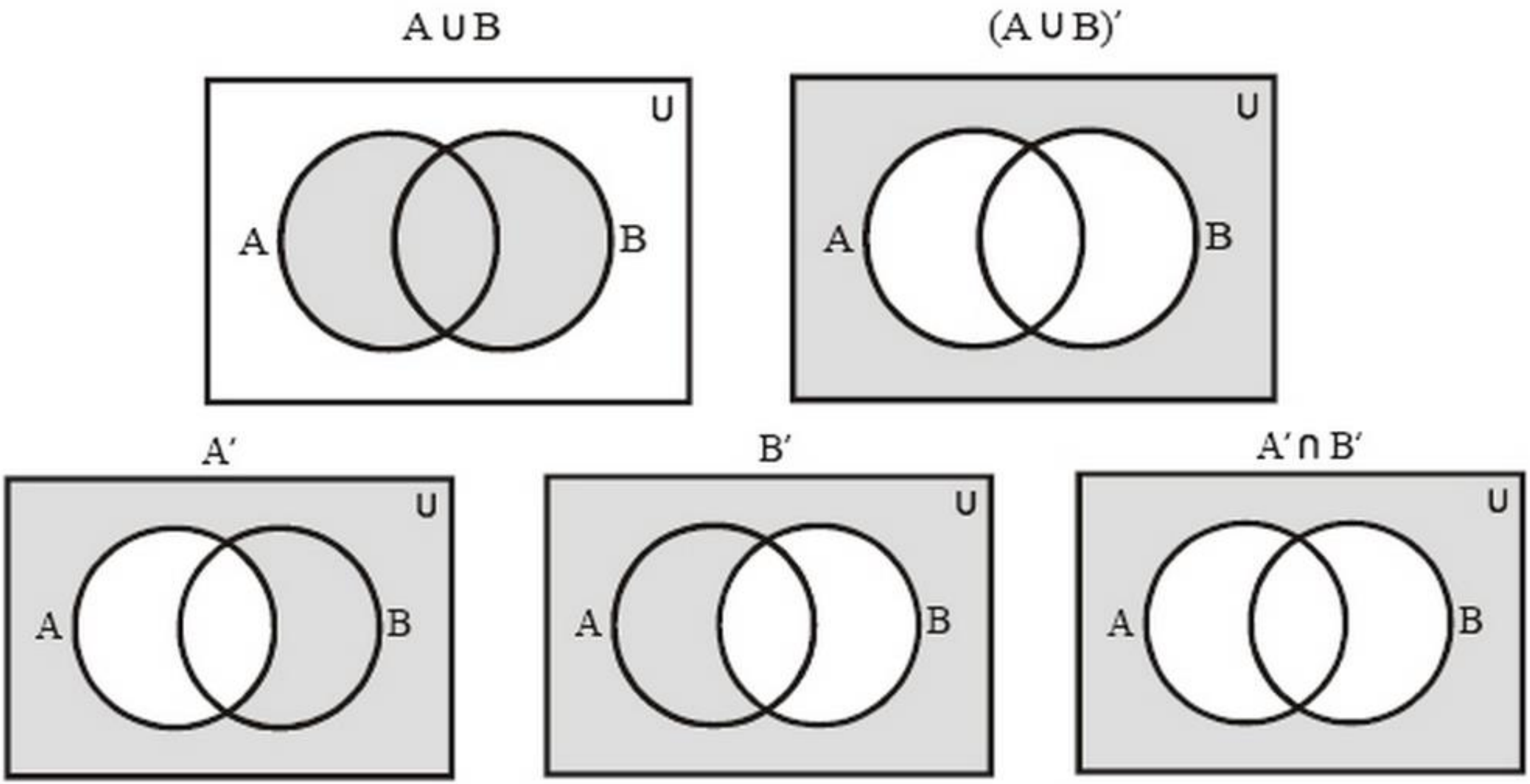
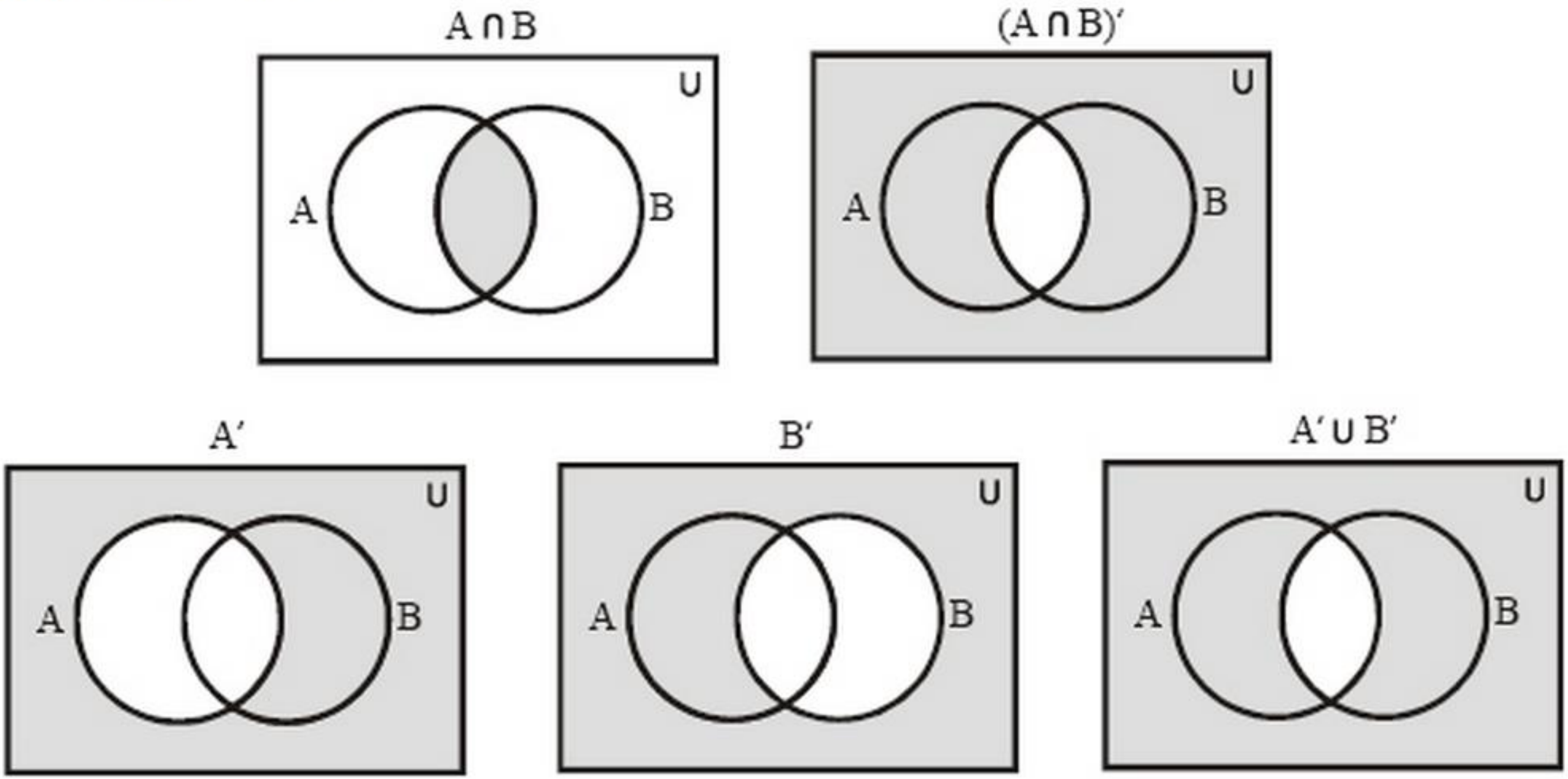
De Morgan Rules

(i) $(A \cup B)' = A' \cap B'$



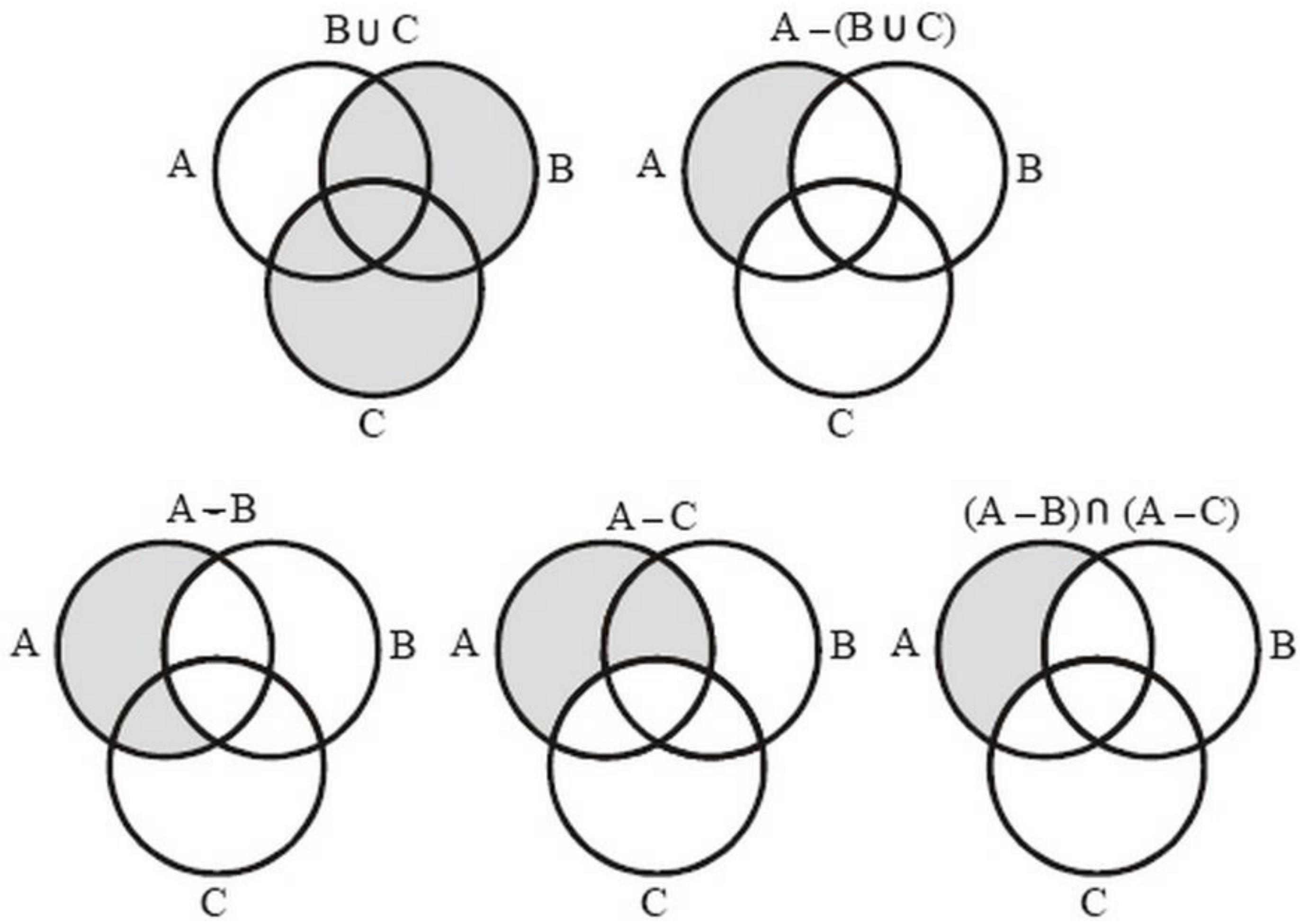
Algebra of Sets

(ii) $(A \cap B)' = A' \cup B'$



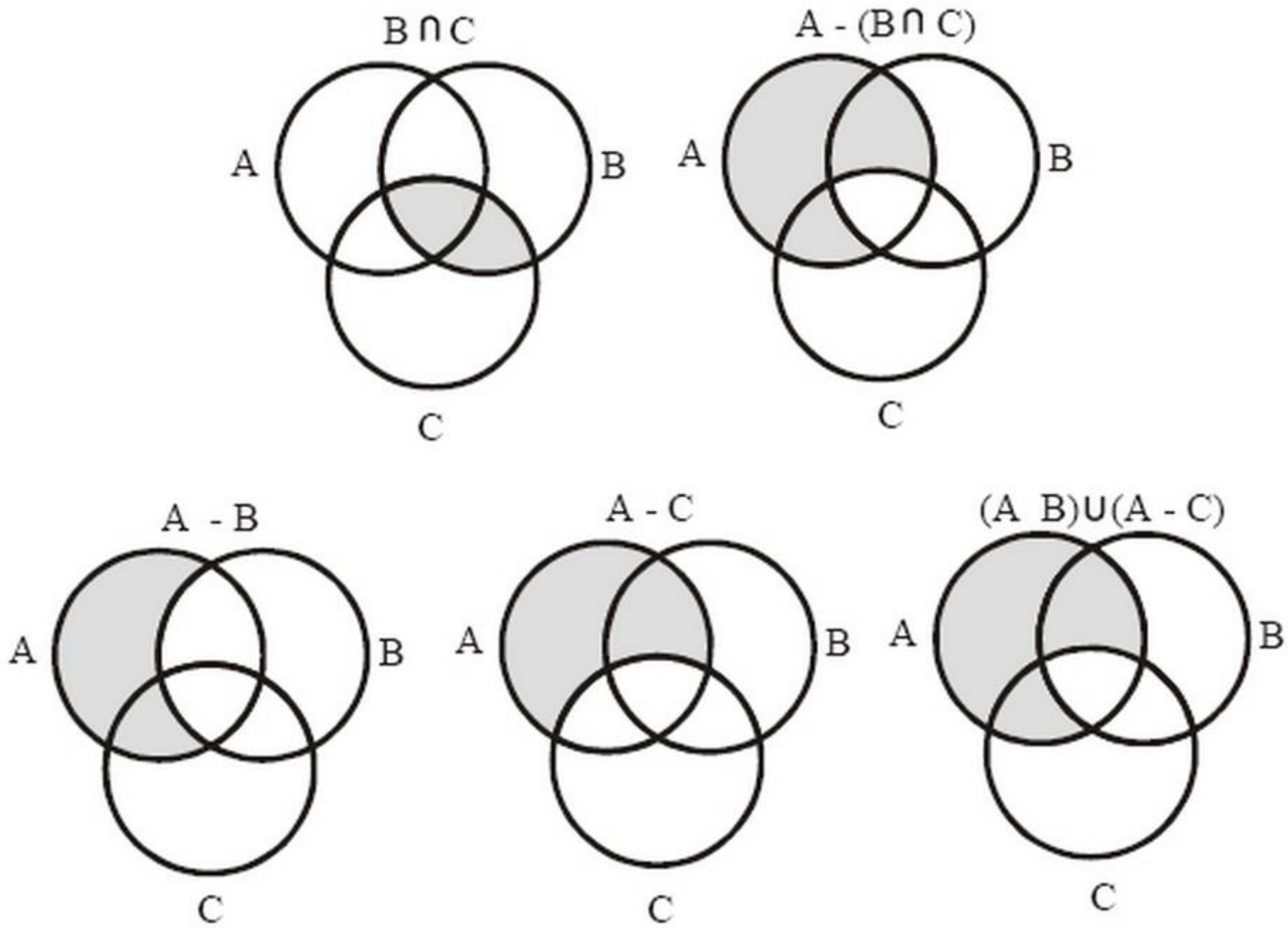
Algebra of Sets

(iii) $A - (B \cup C) = (A - B) \cap A - C)$



Algebra of Sets

(iv) $A - (B \cap C) = (A - B) \cup (A - C)$



Set Identities

Identity	*Name*
$A \cup \varnothing = A$ $A \cap U = A$	Identity laws
$A \cup U = U$ $A \cap \varnothing = \varnothing$	Domination laws
$A \cup A = A$ $A \cap A = A$	Idempotent laws
$\overline{(\overline{A})} = A$	Complementation law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive laws
$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$	De Morgan's laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption laws
$A \cup \overline{A} = U$ $A \cap \overline{A} = \varnothing$	Complement laws

Questions?