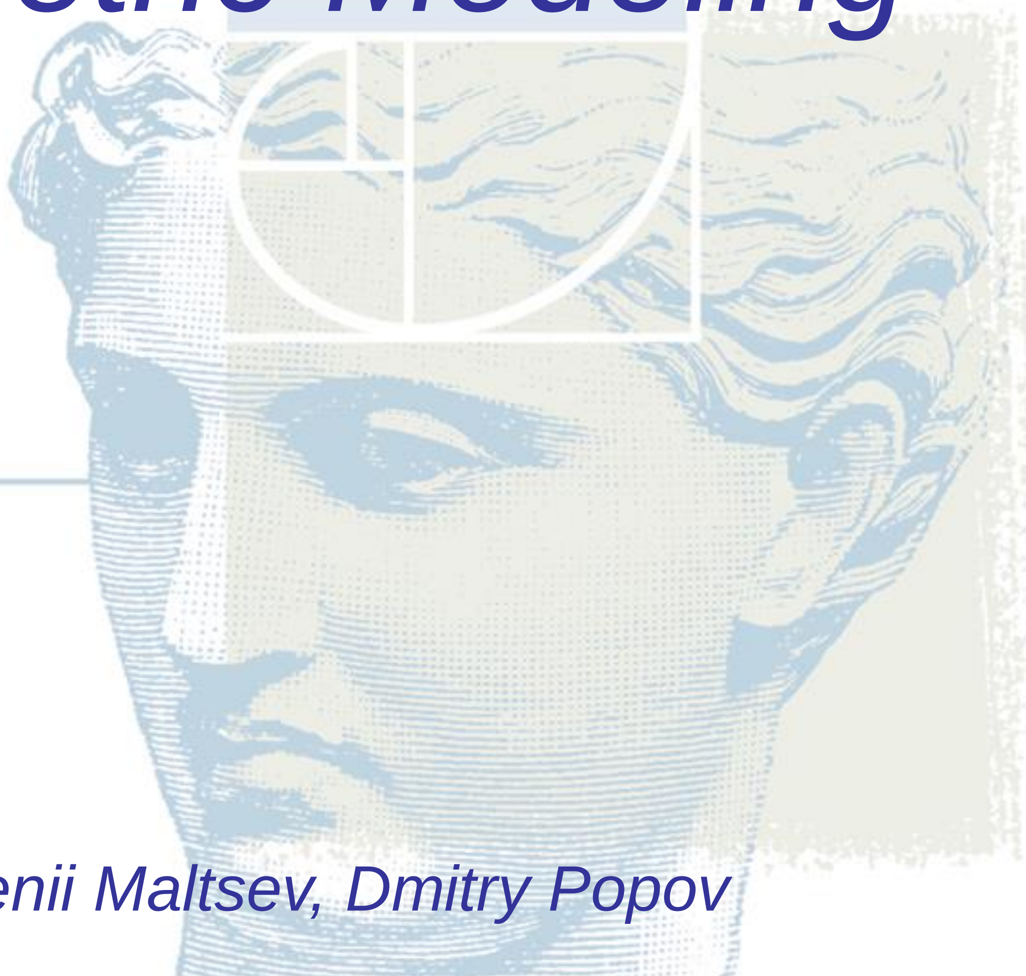
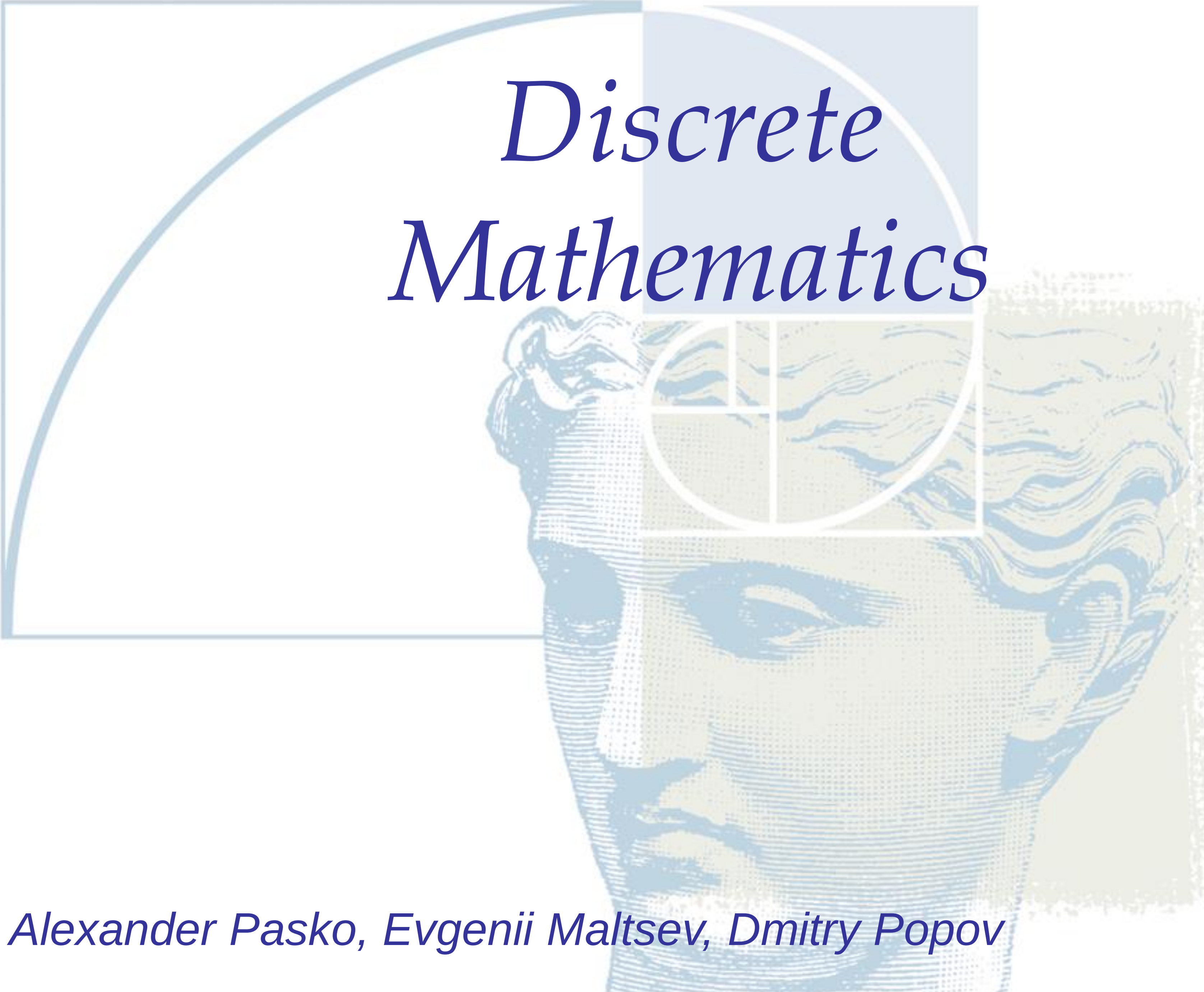
Geometric Modeling



Alexander Pasko, Evgenii Maltsev, Dmitry Popov



Discrete Mathematics

Alexander Pasko, Evgenii Maltsev, Dmitry Popov

Unit materials

- Lecture notes
- Seminar handouts

are available at

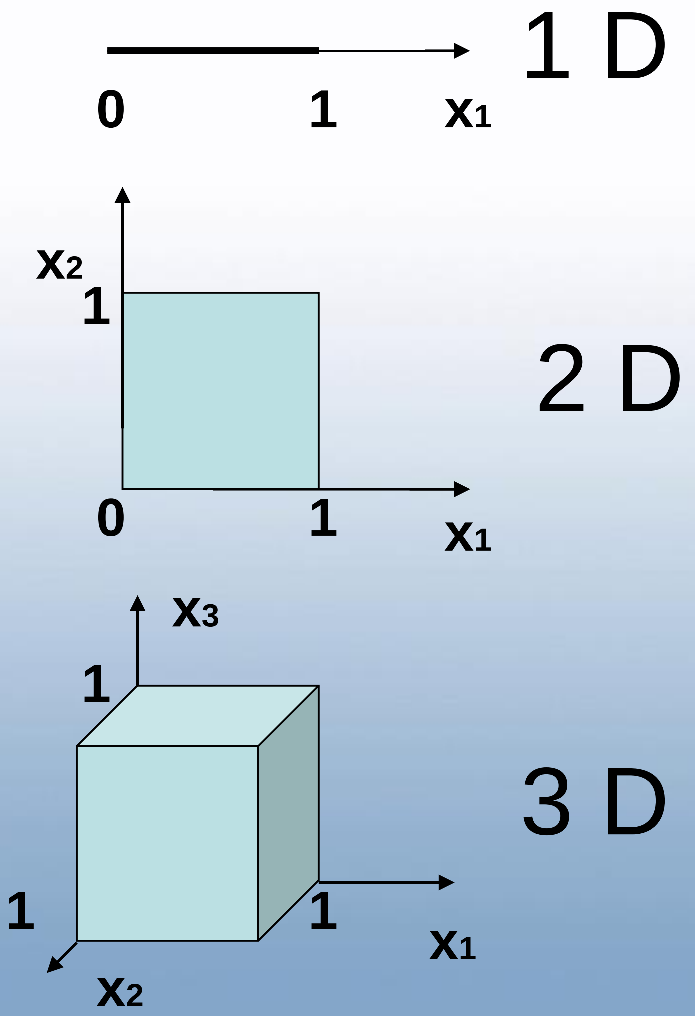
http://gm.softalliance.net/

Advice: download and print lecture notes before the next lecture

Example: k-D unit cube

A unit cube in k-D space is a set of points $P(x_1, x_2, \ldots x_n)$ such as:

$$0 \le x_1 \le 1$$
$$0 \le x_2 \le 1$$
$$\ldots$$
$$0 \le x_n \le 1$$



Shape Dimension

A shape is k-dimensional if there is a continuous one-to-one mapping of the k-dimensional cube (ball) to this shape.

$k \leq n$, $n = 1$ - 4	***Shape***
0	Point
1	Curve
2	Surface
3	Solid
$k = 3$, $n = 4$	Volume

Defining a Point Set

- List of points
- Mapping of a known set
- Point membership rule
- Generation rule

List of Points

2D space	3D space	nD space
$<X_1, Y_1>$	$<X_1, Y_1, Z_1>$	$<X_{11}, X_{12}, X_{13}, \ldots, X_{1n}>$
$<X_2, Y_2>$	$<X_2, Y_2, Z_2>$	$<X_{21}, X_{22}, X_{23}, \ldots, X_{2n}>$
…	…	…
$<X_k, Y_k>$	$<X_k, Y_k, Z_k>$	$<X_{k1}, X_{k2}, X_{k3}, \ldots, X_{kn}>$

Model: Linear array defines one point in nD space

Only finite point sets can be defined in this way and no continuous shape (such as curve or surface) can be defined.

Scanned point cloud

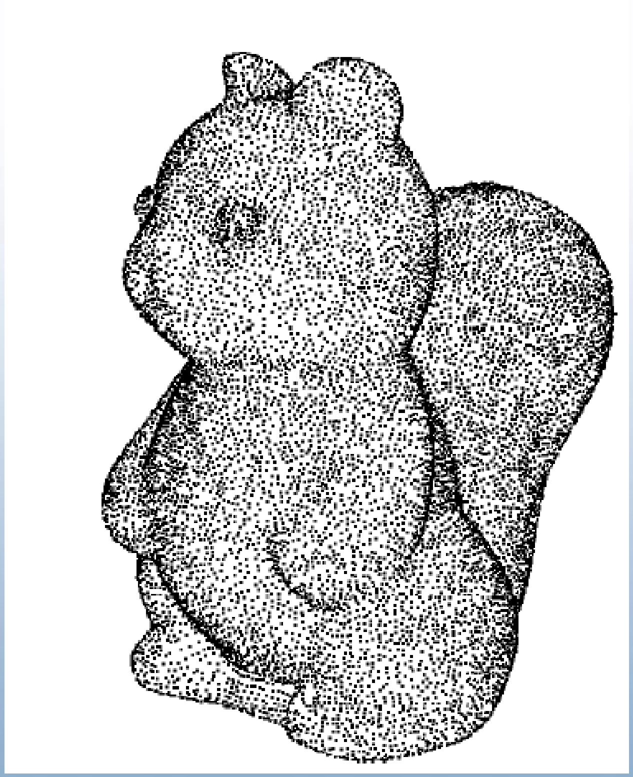
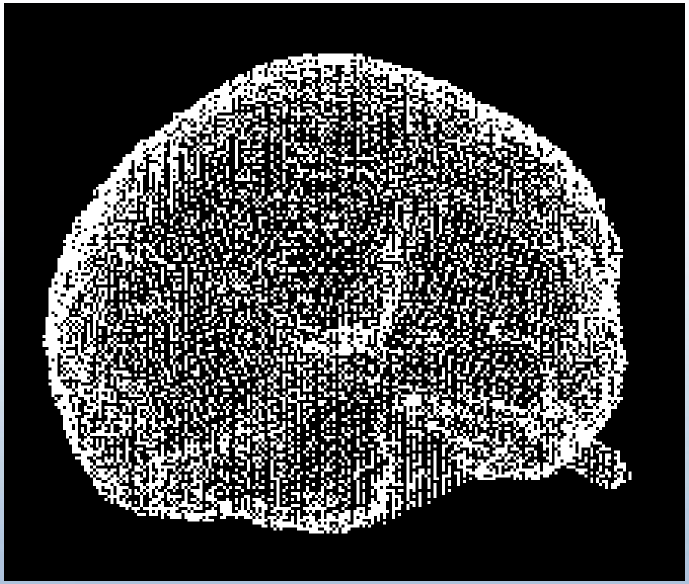


Image by Yu. Otake and A. Belyaev



Point Cloud of a Human Brain

http://www.fpsols.com/point_cloud.html

Examples of Particle systems

Stormy sea

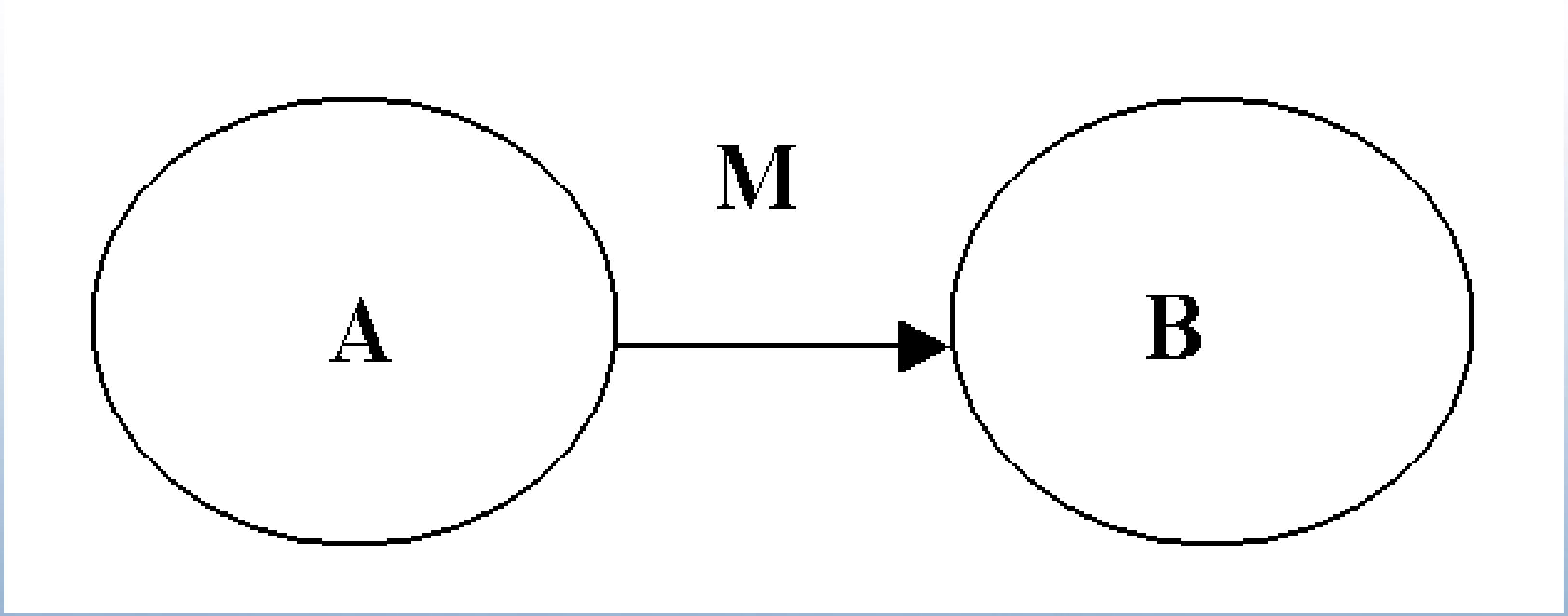
Animation by Steve Green
DreamScape plug-in to 3DS MAX

Explosion

Animation by Thomas Marque
DreamScape plug-in to 3DS MAX

Mapping of a Known Set

$$M \; : \; A \rightarrow B$$



Parametric curves, surfaces and volumes are defined in this way.

“Explicit” Curve in 2D

Mapping

F: R $\rightarrow$ R

Definition:

y = f(x)

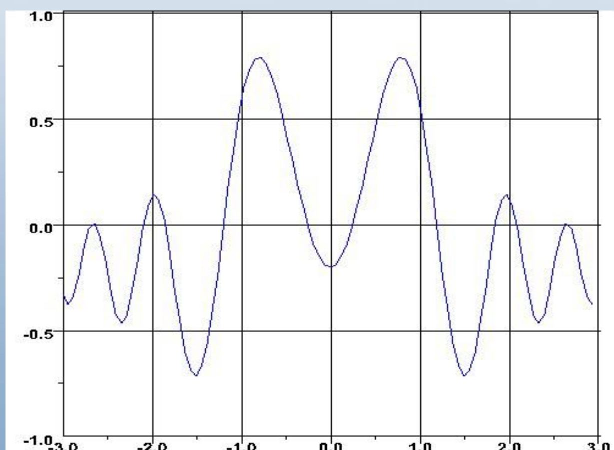


Image from HyperFun

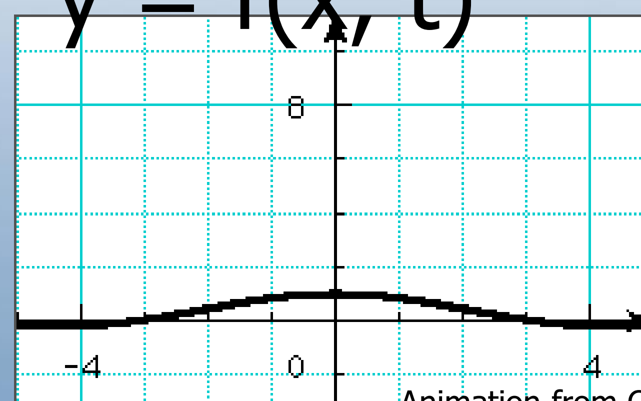
\+ time t

Mapping

F: $R^2 \rightarrow$ R

Definition:

y = f(x, t)



Animation from CurvusPro

“Explicit” Surface in 3D

Mapping

$F: R^2 \rightarrow R$

Definition:

$z = f(x,y)$

+ time t

$F: R^3 \rightarrow R$

Definition:

$z = f(x,y,t)$

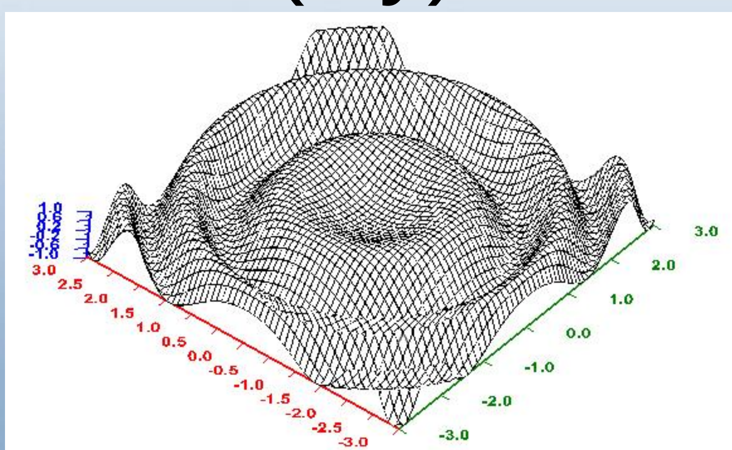
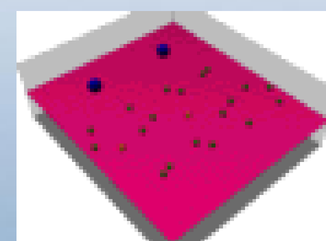


Image from HyperFun



Animation from CurvusPro

Other terms: relief surface, height field, depth field, 2.5D

Volume –
"Explicit" Hypersurface in 4D

Mapping F: $R^3 \rightarrow R$

Definition: $\lambda = f(x,y,z)$

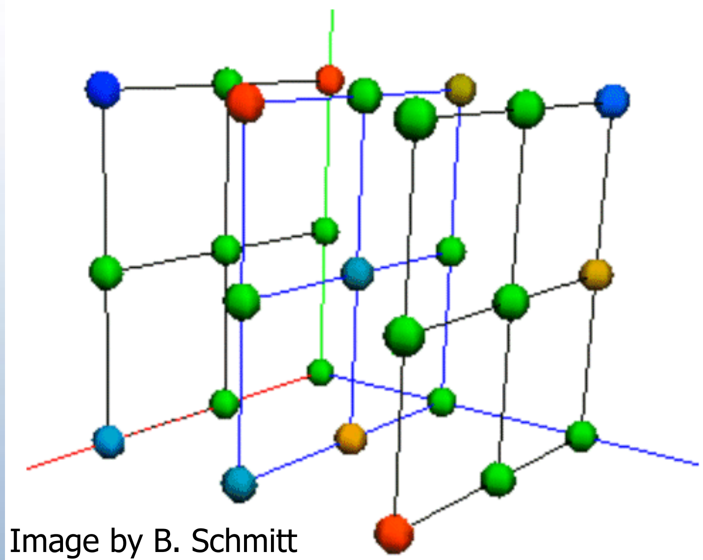


Image by B. Schmitt

Discrete scalar field: function λ is defined in the grid nodes

Volume rendering of smoke density function λ

Image by A. Winter

Other terms: volumetric object, voxel object, 3D scalar field

Volume Image of Head

This example shows a volume rendered as a semitransparent media with variable density in space.

Image by Volume Graphics IPI vlib

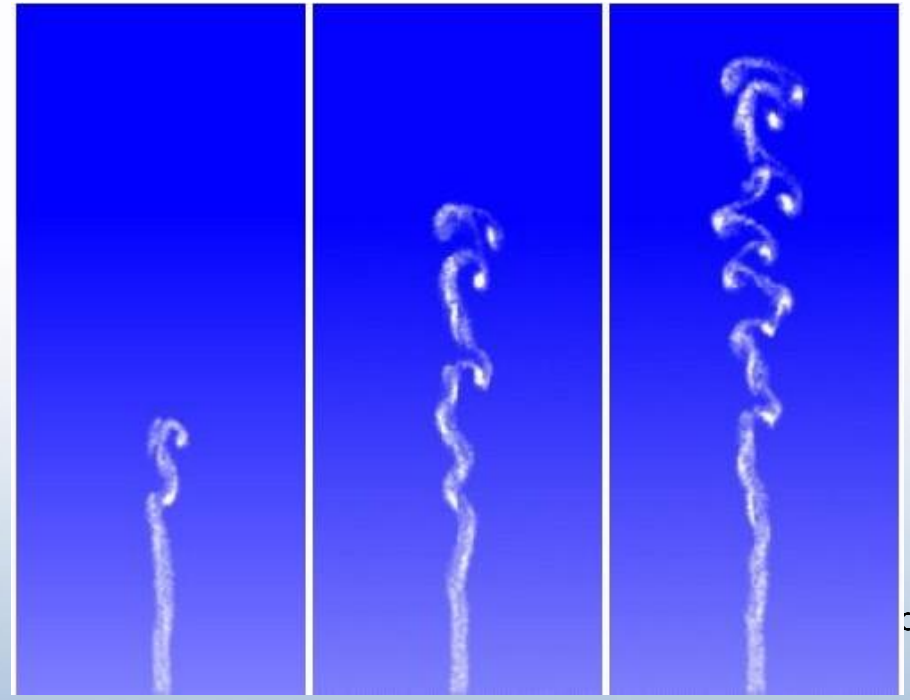
Volume - “Explicit” Hypersurface in 4D

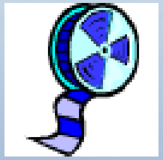
+ time t

Mapping F: $R^4 \rightarrow R$

Definition:

$\lambda = f(x,y,z,t)$





Frames of volumetric animation - rendering of time-dependent smoke density function λ

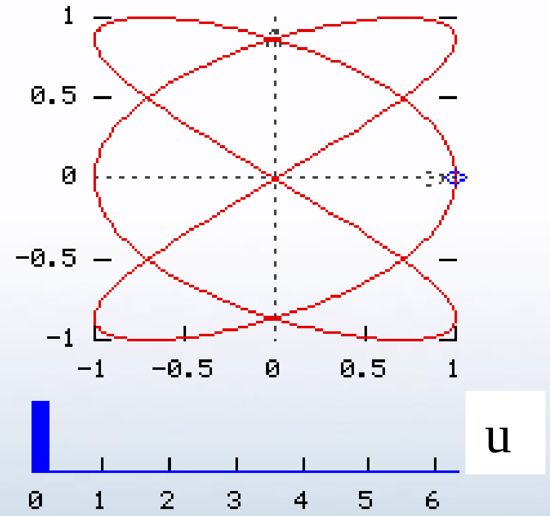
2D Parametric Curve

Mapping F: $R \rightarrow R^2$

Definition:

$$x = x(u)$$

$$y = y(u)$$



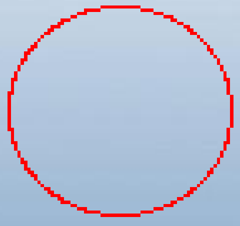
\+ time t

Mapping F: $R^2 \rightarrow R^2$

Definition:

$$x = x(u, t)$$

$$y = y(u, t)$$



Animations from WIMS

at wims.univ-mrs.fr

3D Parametric Curve

Mapping F: $R \rightarrow R^3$

$x = x(u)$

$y = y(u)$

$z = z(u)$

+ time t

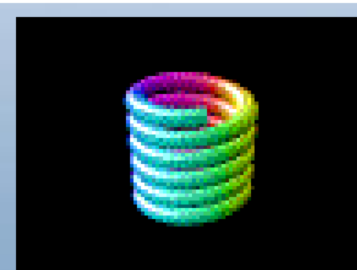
Mapping F: $R^2 \rightarrow R^3$

Definition:

$x = x(u, t)$

$y = y(u, t)$

$z = z(u, t)$



Animations from DPGraph

http://www.dpgraph.com

Parametric curve example

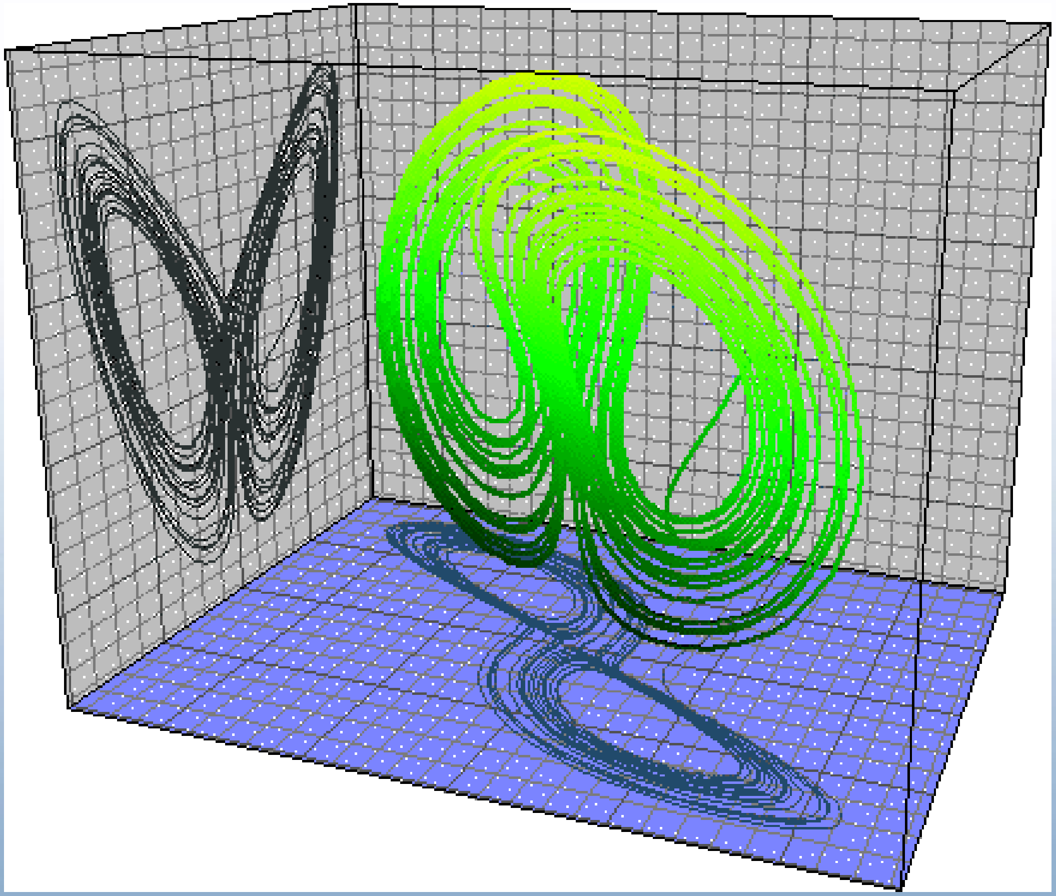
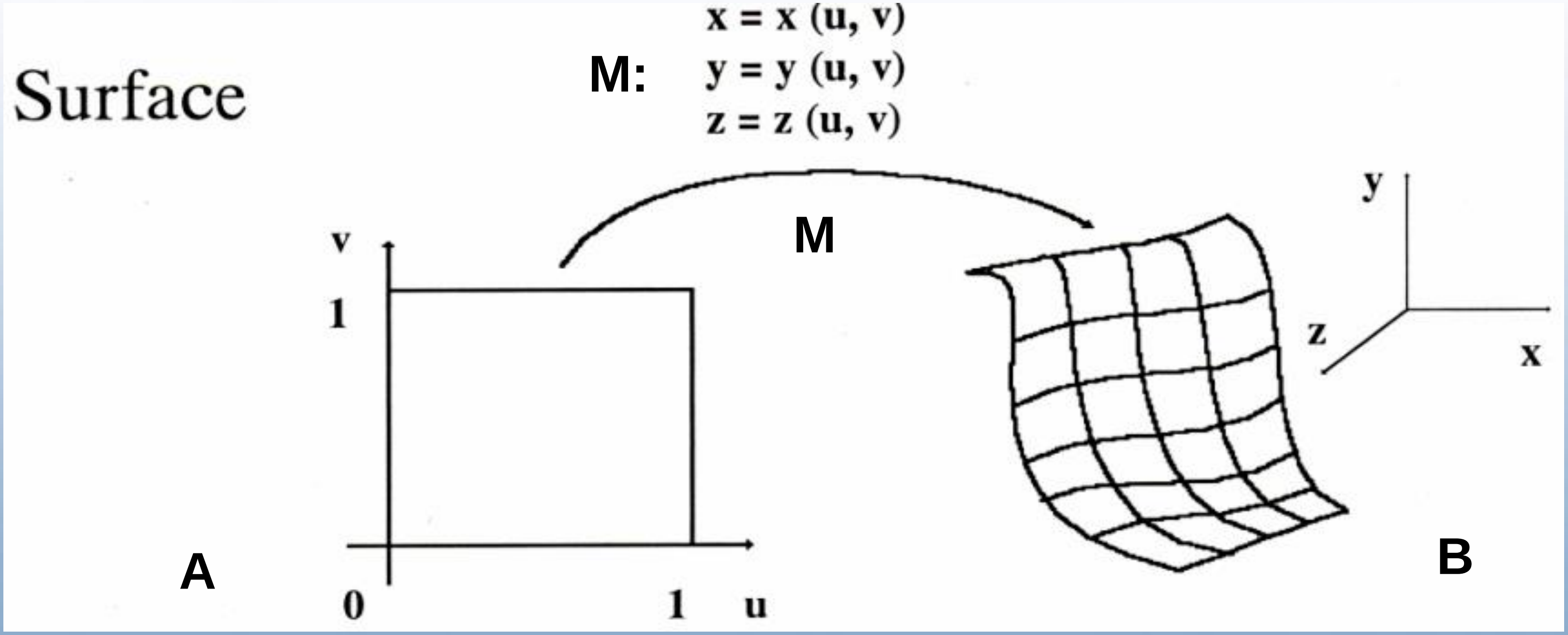


Image from CurvusPro

Parametric Surface

Mapping F: $E^2 \rightarrow E^3$

Model:



Parametric spiral surface

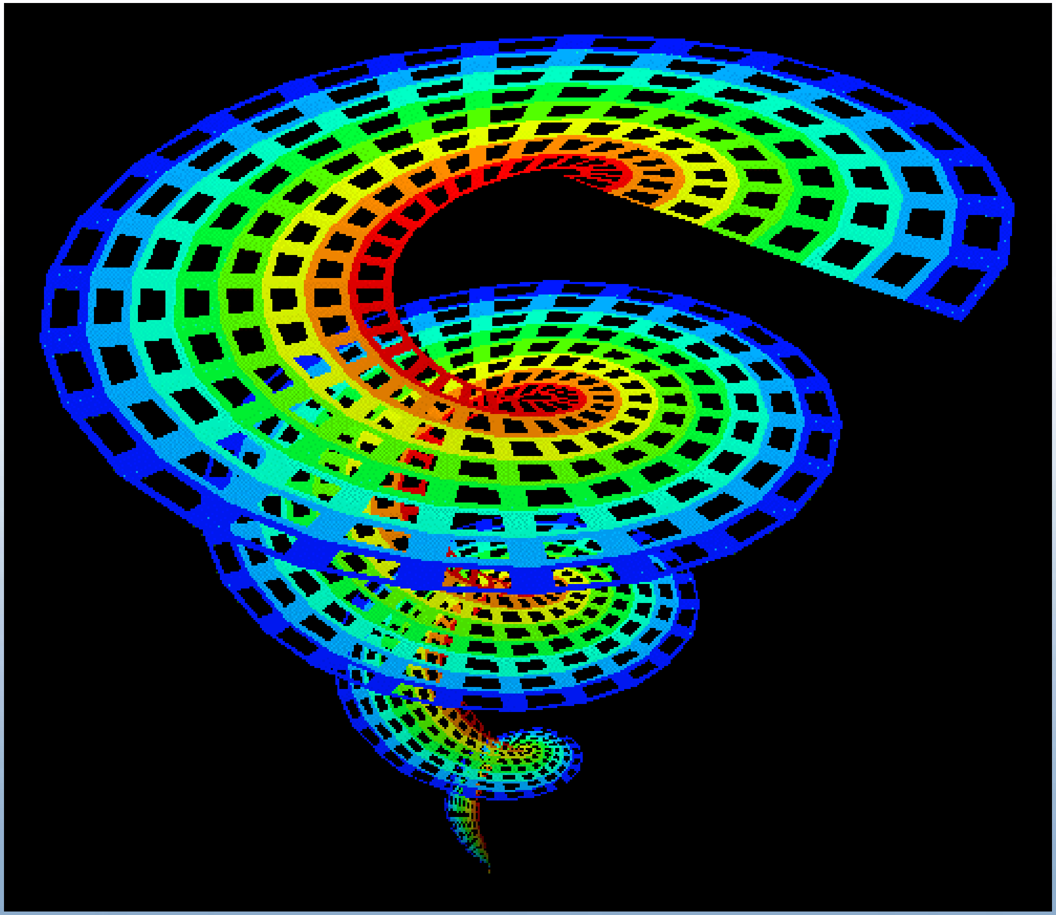


Image from CurvusPro

+ time T

Mapping F: $R^3 \rightarrow R^3$

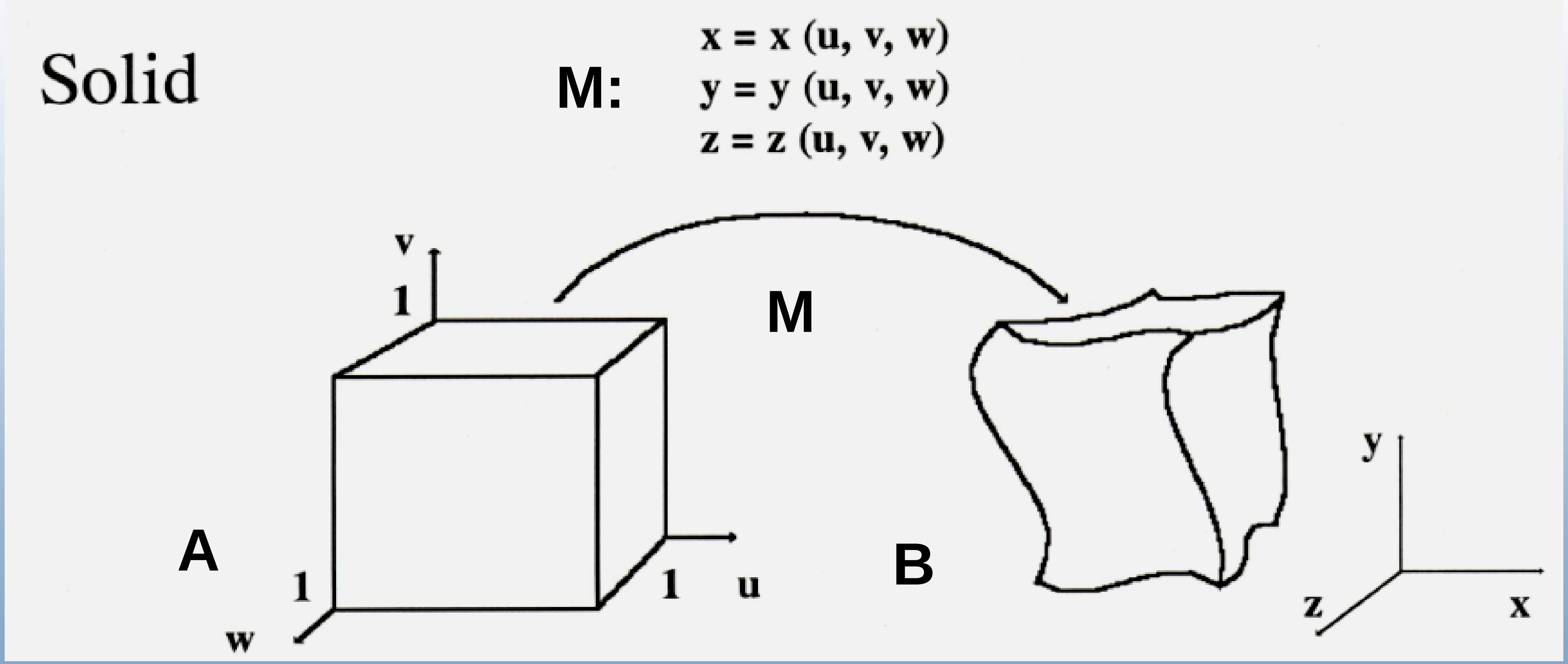
Definition:
$$x = x(u, v, t)$$
$$y = y(u, v, t)$$
$$z = z(u, v, t)$$

Animation by David Parker

Parametric Solid

Mapping F: $E^3 \rightarrow E^3$

Model:



Parametric Coons Solids

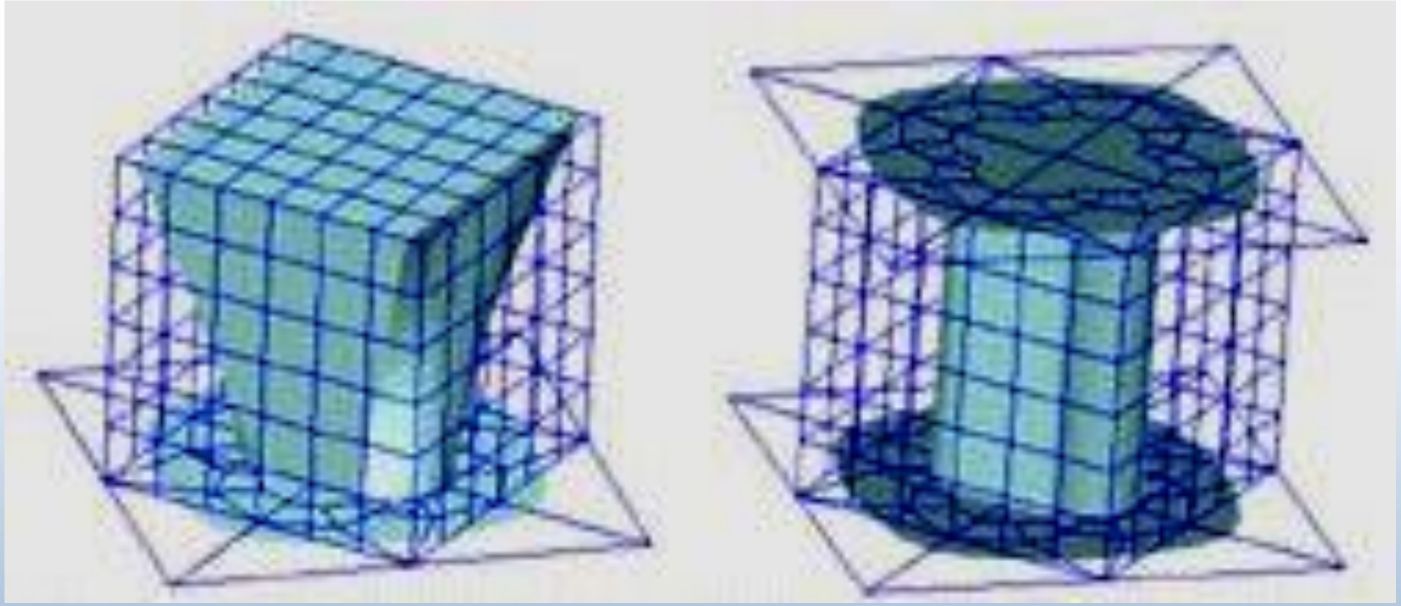


Image by S. Czanner and R. Durikovic, University of Aizu

Point Membership Rule

“Implicit” form

"Implicit" Form

$f(x_1, x_2, \ldots, x_n)$ - continuous real function of n variables.

Implicit objects in nD space:

Solid (k=n):

$f(x_1, x_2, \ldots, x_n) \geq 0$

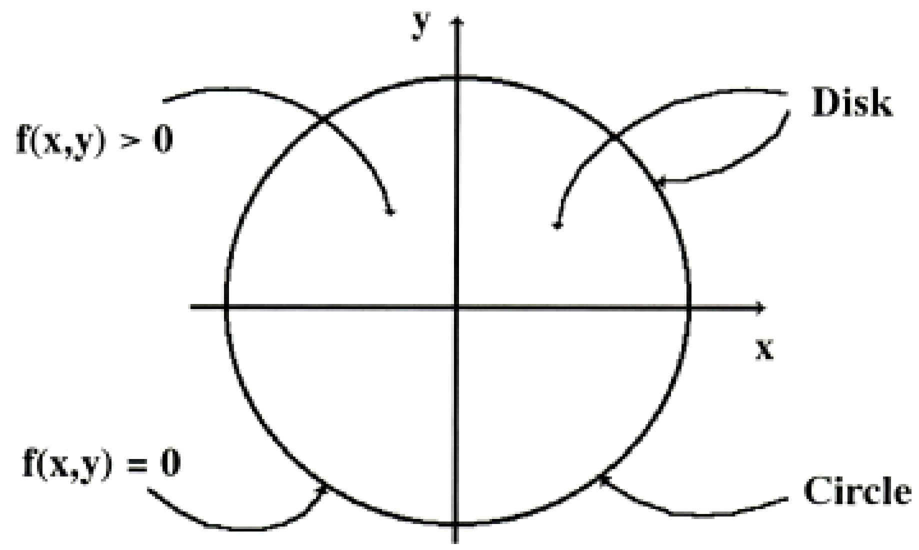
Others (k < n):

$f(x_1, x_2, \ldots, x_n) = 0$

$f(x,y) = R^2 - x^2 - y^2$

Disk (k=2) $f(x,y) \geq 0$

Circle (k=1) $f(x,y) = 0$



“Implicit” Curve in 2D

$f(x,y)=0$

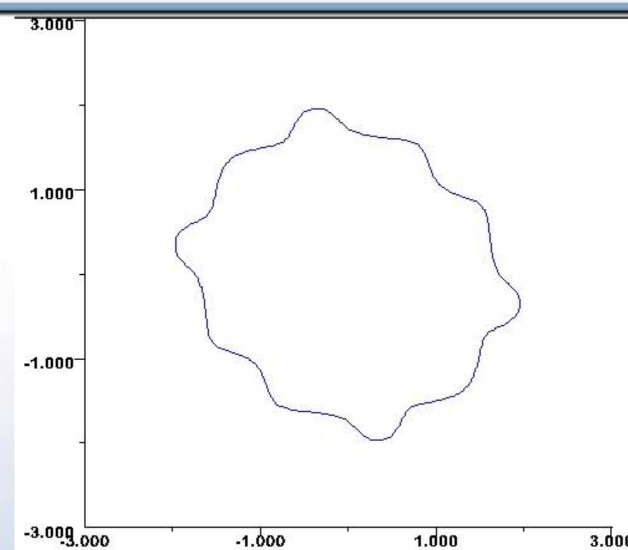
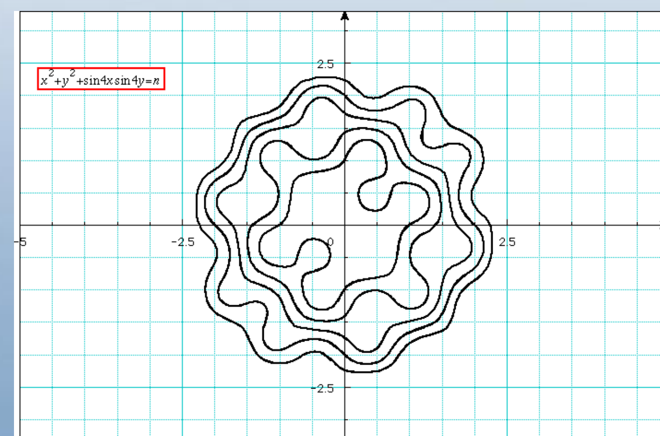


Image from HyperFun

+ time t

$f(x,y,t) = 0$



Animation from HyperFun

Image from CurvusPro

Isosurface or "Implicit" Surface

$$\xi = f(x,y,z)$$

is a function of three variables and a surface

$$\xi = 0 \quad \text{or} \quad f(x,y,z) = 0$$

is an iso-valued surface (*isosurface*) or an "implicit" surface)

Sphere: $R^2 - x^2 - y^2 - z^2 = 0$

Implicit Surfaces and Solids

A set of points in 3D space with

$$f(x,y,z) = 0$$

is called an implicit surface

A 3D solid is defined as

$$f(x, y, z) \geq 0$$

with the implicit surface as its boundary.

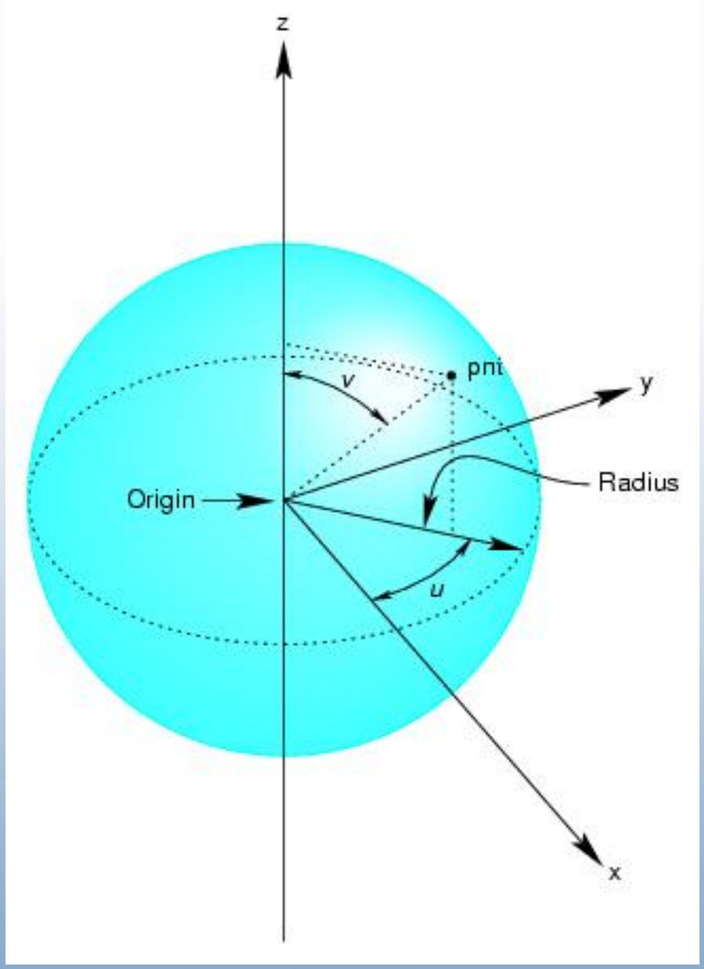
Sphere and Solid Ball

Sphere surface:

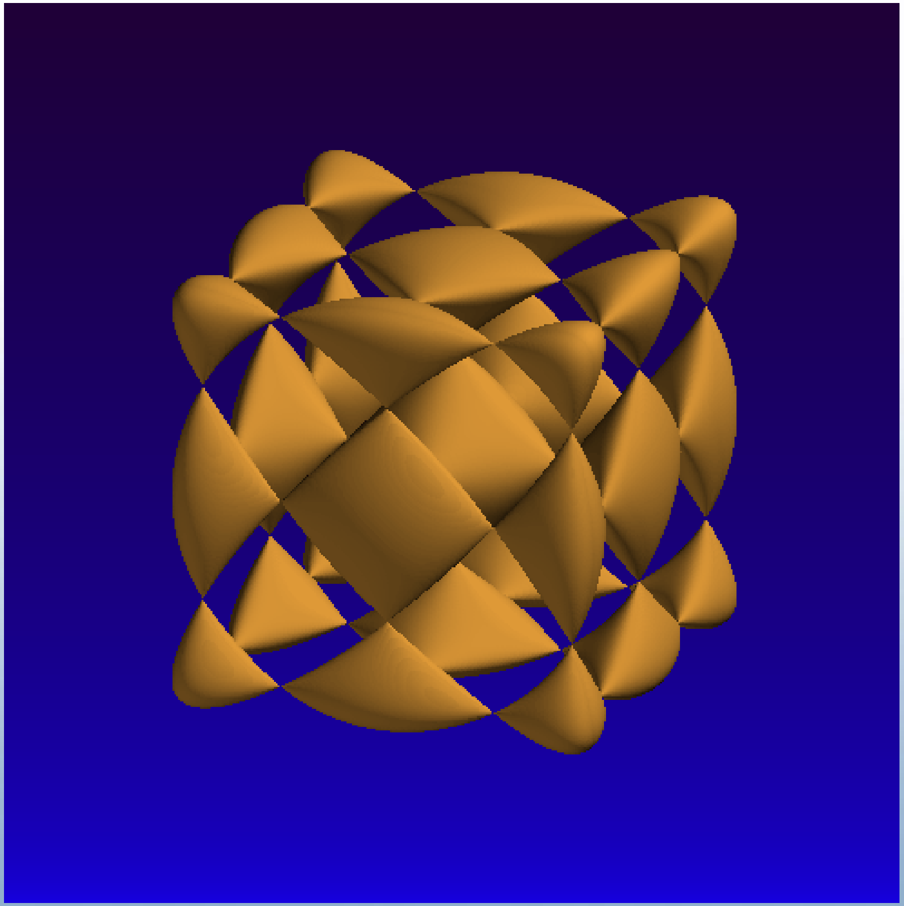
$R^2\text{-}x^2\text{-}y^2\text{-}z^2 = 0$

Solid ball:

$R^2\text{-}x^2\text{-}y^2\text{-}z^2 \geq 0$



Chebyshev Polynomial



Complex isosurface defined by equation f(x,y,z)=0

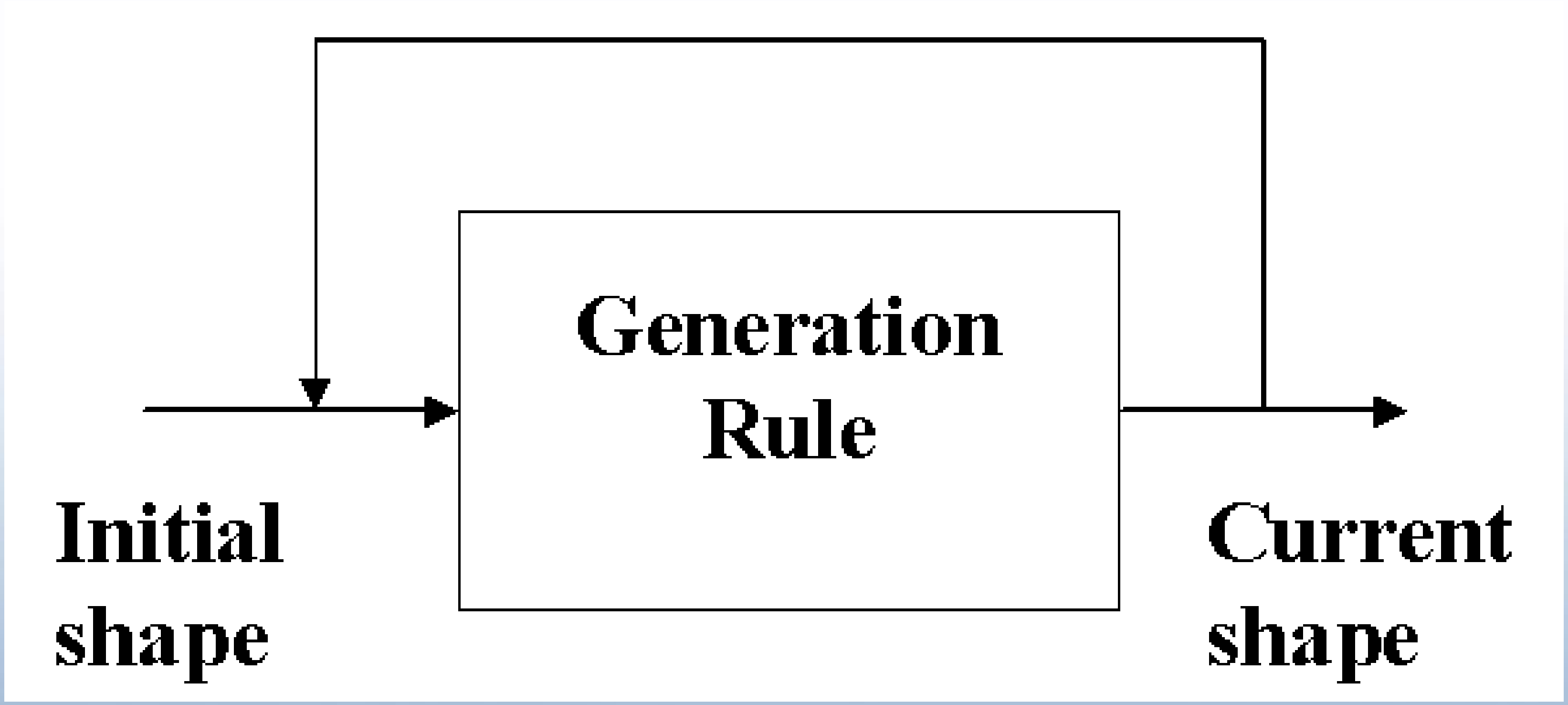
Teeth Isosurface

Isosurface defined by volume data $f(x_i,y_j,z_k)=c$

Image by D. Fang et al.,
University of California, Devis

Dataset of Siemens Medical Systems

Generation Rule



A rule can be specified to generate a shape in a recursive manner (fractals, L-systems, other procedural models)

Fractals

Model:

iterative functions $p' = f(p)$

in 2D or 3D space.

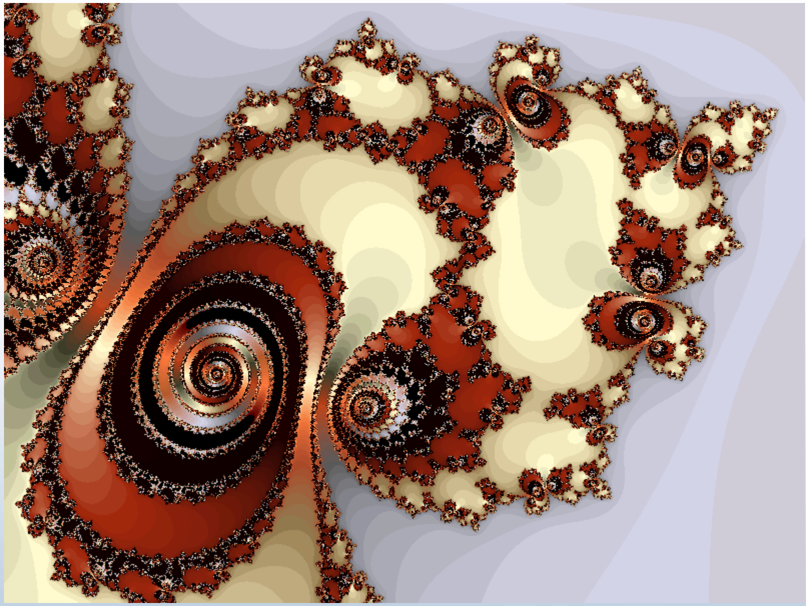


Image by Linda Allison

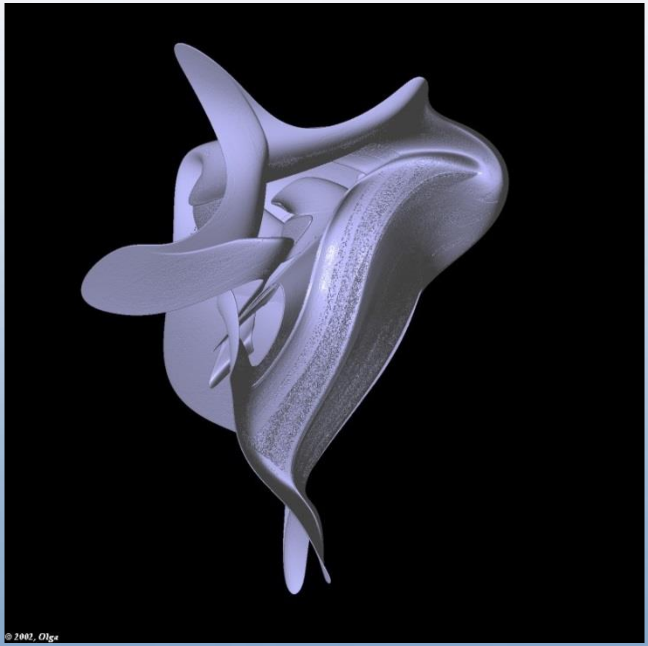


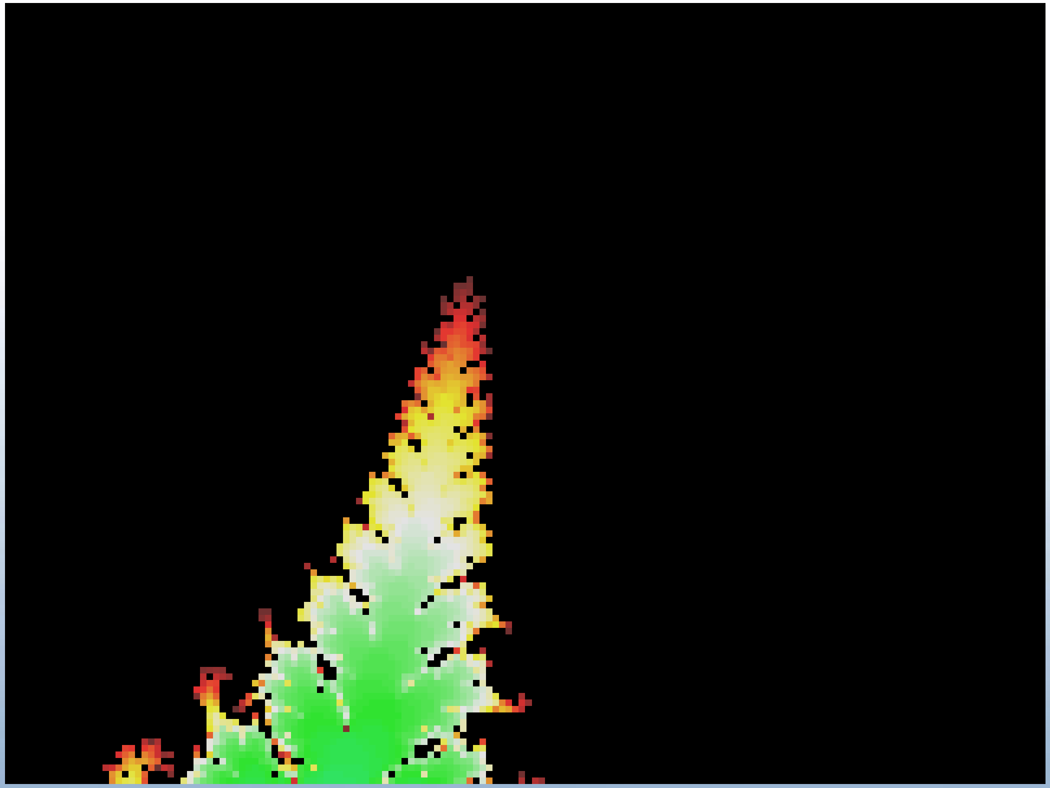
Image “Thick ballerina” by Olga

http://www.eclectasy.com/Fractal-Explorer/

Fractal animation

+ time t

  $p' = f(p,t)$



Animation from Filmer

by Julian Haight

L - systems

Model: grammar

Example:

1) Axiom X
2) Rules

X --> F-[[X]+X]+F[+FX]-X

F --> FF

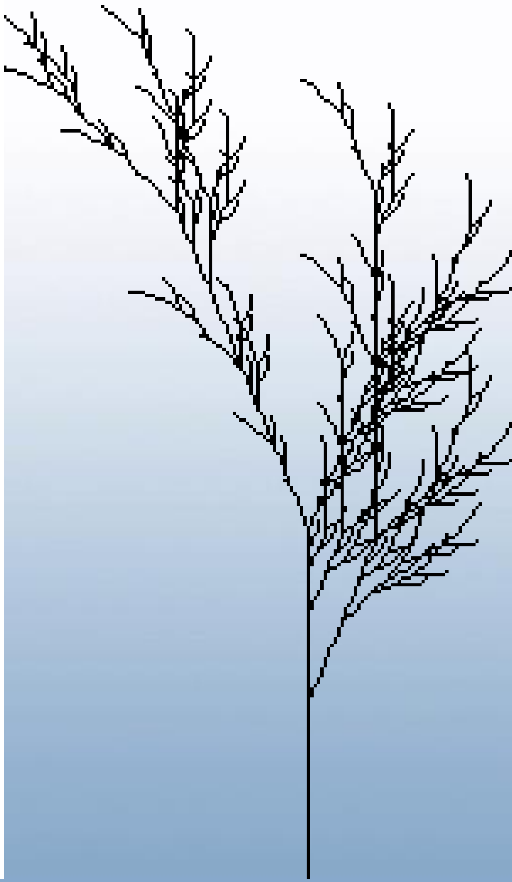


Image by P. Bourke, Swinburne University

Words of wisdom

"Geometry is the mathematical science of shape"

"Without geometry, life is pointless"

www.proudnerd.com

References

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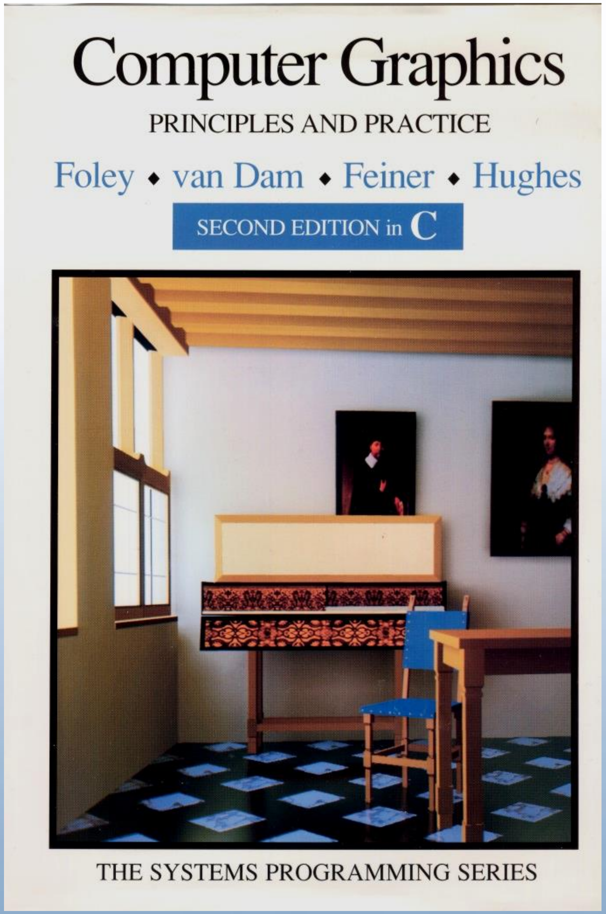

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