# Algorithms of the generation of segments on the plane

**Aim:** The study of the algorithms for the generation of the line segments on the plane.

**Task:** Implement the software for drawing of a shape on the plane with using the asymmetric digital differential analyzer and Bresenham's line algorithms.

**Result:** The executed binary file. The source code. The report.

## Theoretical part:

The task of drawing of line segments is not so easy as would seem. Main task of such algorithms is calculation of the pixel coordinates which lay as close to the real segment line as possible. There are several algorithms for generation of line segments. Let’s consider two incremental algorithm: asymmetric digital differential analyzer and Bresenham's line algorithm

### *Asymmetric digital differential analyzer (ADDA)*

This algorithm is based on solution of differential equation (1):

$$\mathrm{D} = \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\Delta x}{\Delta \mathrm{y}} = \frac{x_1 - x_2}{y_1 - y_2} \qquad (1)$$

This is derivative of our function, which is straight line, and this quantity is constant. The derivative is used for the calculation one coordinate of next point when another is incremented by 1. (Fig.1.1)

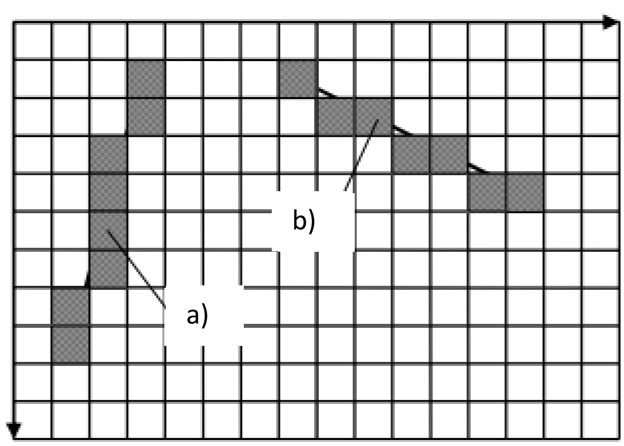



Fig 1.1 Algorithm for asymmetric digital differential analyzer

Algorithm:

1. Calculate D for definition of direction of way (X or Y):
   a. If $\Delta x = 0$ then the line segment is vertical, D = 0. Y coordinate will be incremented.
   b. If $\Delta y = 0$ then the line segment is horizontal, D = 0. X coordinate will be incremented.
   c. If $\Delta x \geq \Delta y$ then $D = \frac{\Delta x}{\Delta y}$, X coordinate will be incremented.
   d. If $\Delta x < \Delta y$ then $D = \frac{\Delta y}{\Delta x}$, Y coordinate will be incremented.
2. Draw pixel with coordinates $(x_1, y_1)$.
3. Increment corresponded coordinate (X or Y) by 1 (-1), and other coordinate incremented by D.
4. Repeat 2-3
5. End

## *Bresenham's line algorithm*

We need to draw a line between points A and B. Start point of drawing is A( *Ax, Ay* ). Let's specify the increment of the variable *Ay* equal 1 in loop and *Ax* stays constant or increases with step 1. Main condition is that new point should be as close to straight line AB as possible (Fig 2.1).

It means that the horizontally distance between selected point and straight line AB should not exceed value 0.5. If derivative < 0.5 then next point will be (1,0) (fig. 2.1a), but if the derivative > 0.5 next point would be (1,1) (fig. 2.1b).

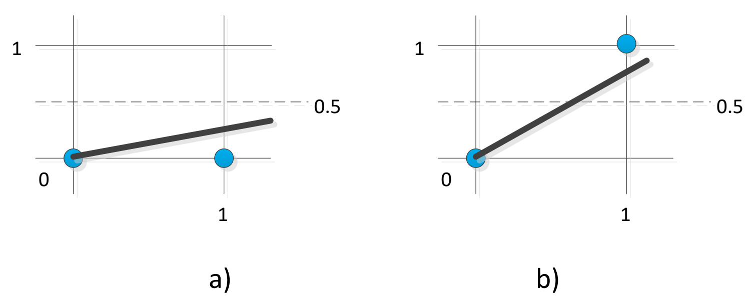



Fig 2.1 Example of segment line

Let's introduce a variable *d* with condition $-0,5 < d \leq 0,5$. This inequality maintains the condition for the increment of variable *Ax*. The sign of *d* is used for choice of closest point.

Example:

The segment line passes throw points (0,0) and (4,1.5). (fig. 2.2) In our case the derivative $\frac{\Delta y}{\Delta x} = 0.3$.

$$\text{Step1: } d_1 = \frac{\Delta y}{\Delta x} - 0.5 = -0.2 < 0 \;\; => \textit{next point (1,0)}$$

Step2: $d_2 = d_1 + \frac{\Delta y}{\Delta x} = 0.1 > 0$ => *next point (2,1)*

Step3: $d_2 = d_2 - 1;\ d_3 = d_2 + \frac{\Delta y}{\Delta x} = -0.6 < 0$ => *next point (3,1)*

Let's do this equation $d_1 = \frac{\Delta y}{\Delta x} - 0.5$ simple by multiplication this equation by 2 *Δx =>

$d_1 = 2*\Delta y - \Delta x$

if ($d_1 > 0$) $d_2 = d_1 + 2*(\Delta y - \Delta x)$

if ($d_1 \leq 0$) $d_2 = d_1 + 2*\Delta y$

So, we get algorithm with integer numbers, subtraction and multiplication by 2.

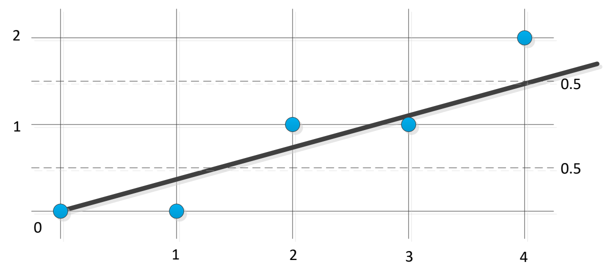



Fig 2.2 Example of segment line

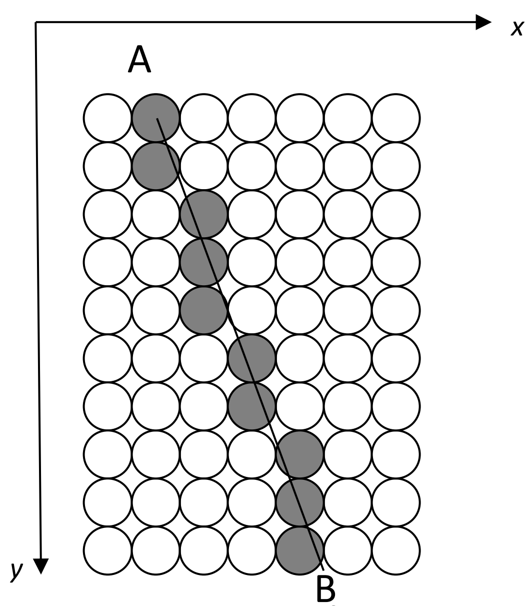



Fig 2.3 Bresenham's line algorithm for version when $\Delta y \geq \Delta x$

Bresenham's line algorithm for version when $\Delta y < \Delta x$

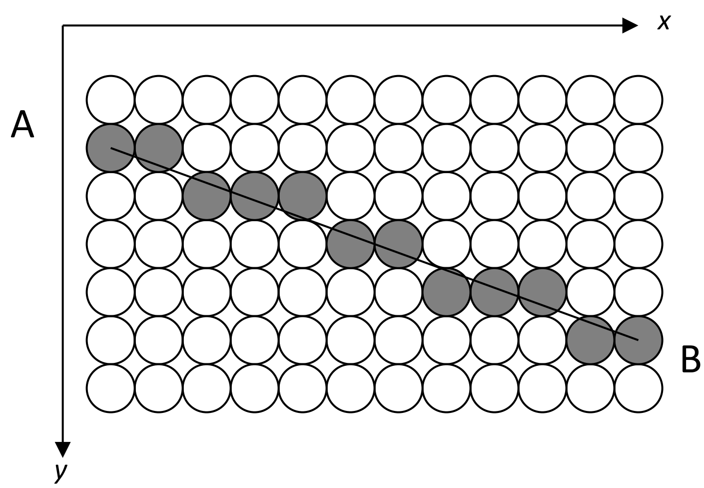



Fig 2.4 Bresenham's line algorithm for version when $\Delta y < \Delta x$

Algorithm:

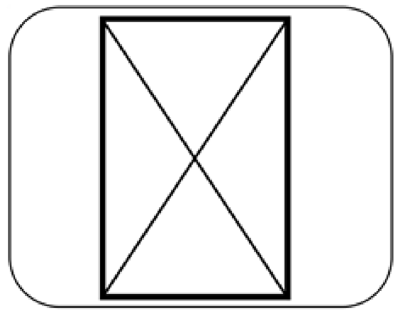
1. Sort of the vertices A and B with condition $Ay \le By$.
2. Calculate values $\Delta x = Bx - Ax$ , and $\Delta y = By - Ay$ .
3. If $\Delta x \ge 0$ than $dx = 1$ else $dx = -1$, and invert sign of $\Delta x$
4. Set $d = 0$. If $\Delta y \ge \Delta x$, that set $t = 2\Delta x$ , $\Delta = 2\Delta y$. Else $t = 2\Delta y$ , $\Delta = 2\Delta x$
5. If $\Delta y \ge \Delta x$ then goto 6 else goto 11.
6. Draw pixel with coordinates (Ax, Ay).
7. $Ay = Ay + 1$; $d = d + t$ .
8. If $d > \Delta y$ then $Ax = Ax + dx$; $d = d - \Delta$;
9. Repeat 6-8
10. Goto End
11. Draw pixel with coordinates (Ax, Ay).
12. $Ax = Ax + dx$; $d = d + t$ .
13. If $d > \Delta x$ then $Ay = Ay + 1$; $d = d - \Delta$
14. Repeat 11-13
15. End

## Tasks:

Using the Bresenham's line algorithm or the ADDA draw your version of figure.

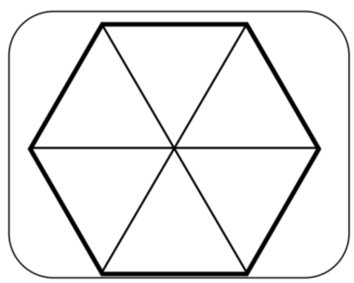
**<u>Exercise 1 :</u>**

Draw rectangle with diagonals that fits in the output area.



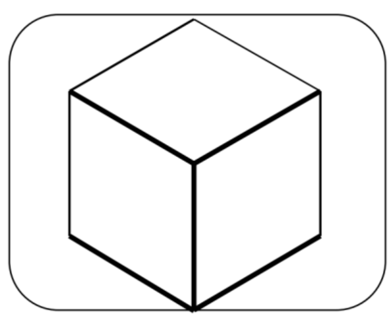
**<u>Exercise 2 :</u>**

Draw hexagon that fits in the output area.



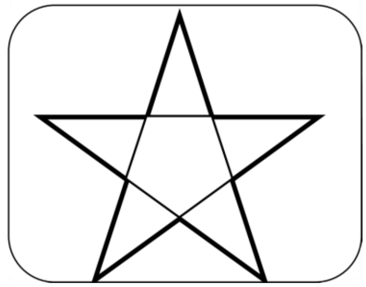
**<u>Exercise 3 :</u>**

Draw cube that fits in the output area.



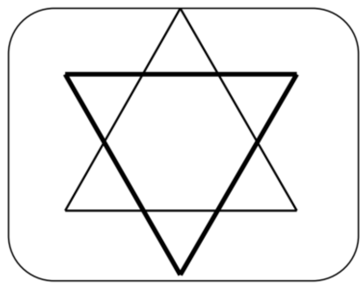
**<u>Exercise 4 :</u>**

Draw 5-pointed star that fits in the output area.



**Exercise 5 :**

Draw 6-pointed star that fits in the output area.



**Exercise 6 :**

Draw octahedron that fits in the output area.